You have a list that contains the integers 101 to 200 (inclusive). You decide to play a game where you pick any two of the numbers on the list, say $a$ and $b$, cross them out, and replace them by a single number $ab + a + b$. You will continue doing this until only one number remains (which will happen after 99 iterations). What will be the last number?

Solution

If a pair of numbers $(a, b)$ on the list are replaced by $ab + a + b$, the product of all the numbers on the list, each incremented by 1, remains unchanged since 

$$(a + 1)(b + 1) = ((ab + a + b) + 1).$$

It follows that, if $N$ denotes the last surviving number, then $N + 1$ is equal to the product of the original 100 numbers, each incremented by 1:

$$N + 1 = \prod_{k=101}^{200} (k + 1) = \frac{201!}{101!}$$

Therefore:

$$N = \prod_{k=101}^{200} (k + 1) = \frac{201!}{101!} - 1$$