Find the number of ordered triples \((G_1, G_2, G_3)\) of subsets of \(\{1, 2, \ldots, 2016\}\) with the following properties:

1. Each of the integers 1, 2, \ldots, 2016 belongs to at least one of the sets \(G_1, G_2, G_3\).
2. None of the integers 1, 2, \ldots, 2016 belongs to all three of the sets \(G_1, G_2, G_3\).

Solution

Encode the memberships of an element in \(\{1, 2, \ldots, 2016\}\) as a triple \((g_1, g_2, g_3)\) with \(g_i = 1\) if the integer belongs to \(G_i\), and 0 otherwise. For each element, there are 6 such encodings that satisfy the both constraints: \((1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0), (0, 1, 1), (1, 0, 1)\). Since there are 2016 elements, the total number of possible encodings, and hence total number of set triples \((G_1, G_2, G_3)\) satisfying both constraints is \(6^{2016} \approx 5.66 \times 10^{1568}\)