These prisoners will get together and count how many of them there are, N. One prisoner is the designated counter. If he (WLOG my prisoners are all male) sees that the switch is up, he switches it down and ups his count by 1. He starts at 0. All of the other prisoners will touch the switch only once. If they go in the room and see that the switch is down, the first time they will push the switch up. Besides that, they will do nothing.

In other words, the switch will go up and down exactly N-1 times. When Mr. Counter's count is at N-1, he will say everyone has been let in once.

What is the expected number of days until somebody flips the switch the first time? This is equal to the number of days until prisoners 1, 2, ..., or N-1 go in the room. That first prisoner is named prisoner 1.

We add up all of the (number of days) * (probability that it will take that number of days) and we get.

\[
1 \cdot \frac{N-1}{N} + 2 \cdot \frac{1}{N} \cdot \frac{N-1}{N} + 3 \cdot \left(\frac{1}{N}\right)^2 \cdot \frac{N-1}{N} + \ldots
\]

\[
\frac{N-1}{N} \sum_{i=1}^{N} \left(\frac{1}{N}\right)^{i-1}
\]

\[
\frac{N}{1 - \frac{1}{N}}
\]

What is the expected number of days until prisoner N is chosen?

\[
1 \cdot \frac{1}{N} + 2 \cdot \frac{N-1}{N} \cdot \frac{1}{N} + 3 \cdot \left(\frac{N-1}{N}\right)^2 \cdot \frac{1}{N} + \ldots
\]

\[
\frac{1}{N} \sum_{i=1}^{N} \left(\frac{N-1}{N}\right)^{i-1}
\]

\[
\frac{N}{1 - \frac{1}{N}}
\]

What is the expected number of days until the switch gets flipped for the second time? This is equal to the number of days until prisoners 2, 3, ..., or N-1 go in the room. That second prisoner is named prisoner 2.

\[
\frac{N-2}{N} \sum_{i=1}^{N} \left(\frac{2}{N}\right)^{i-1}
\]

\[
\frac{N}{1 - \frac{2}{N}}
\]
What is the expected number of days until the switch gets flipped for the kth time? This is equal to the number of days until prisoners k, k+1, ..., or N-1 go in the room. That kth prisoner is named prisoner k.

\[
\text{Simplify} \left[ \frac{N-k}{N} \sum_{i=1}^{N-1} \left( \frac{k}{N} \right)^{i-1} \right]
\]

\[
\frac{N}{k-N}
\]

How long until the switch gets flipped up N-1 times and then flipped down N-1 times? I add up all the expected values. I can't get a general formula to work, so I included a table and graph.

\[
\text{Table} \left[ N \times (N-1.) + \sum_{k=1}^{N-1} \frac{N}{N-k}, \{N, 2, 30\} \right]
\]


\[
\text{ListPlot} \left[ \text{Table} \left[ N \times (N-1.) + \sum_{k=1}^{N-1} \frac{N}{N-k}, \{N, 1, 30\} \right] \right]
\]