“Problem 9: Nines”

There are \( \binom{4}{2} \cdot \binom{48}{2} \) ways to draw two 9s in a hand of four cards out of a fifty-two card deck, and there are \( \binom{52}{4} \) possible hands. Therefore the probability of drawing two 9s in a hand of four cards is

\[
\frac{\binom{4}{2} \cdot \binom{48}{2}}{\binom{52}{4}} = \frac{6768}{270725} \approx 0.0249995. \tag{1}
\]

This leaves two 9s left for the flop, and so the probability of having the two 9s appear in the three card flop (given your hand with a pair of 9s) is

\[
\frac{\binom{2}{2} \cdot \binom{46}{1}}{\binom{48}{3}} = \frac{1}{376} \approx 0.00265957. \tag{2}
\]

Multiplying equations (1) and (2), we obtain the probability of being dealt a pair of 9s and having the other two 9s show up in the flop,

\[
\left( \frac{\binom{4}{2} \cdot \binom{48}{2}}{\binom{52}{4}} \right) \cdot \left( \frac{\binom{2}{2} \cdot \binom{46}{1}}{\binom{48}{3}} \right) \cdot \frac{6768}{270725} \cdot \frac{1}{376} = \frac{18}{270725} \approx 6.64881 \times 10^{-5}.
\]

Assuming the deck is well-shuffled between the two hands (so that the cards dealt in one hand don’t affect those dealt in the next), the probability of getting two such hands in a row is,

\[
\left[ \frac{\binom{4}{2} \cdot \binom{48}{2}}{\binom{52}{4}} \cdot \frac{\binom{2}{2} \cdot \binom{46}{1}}{\binom{48}{3}} \right]^2 = \left( \frac{18}{270725} \right)^2 = \frac{324}{73,292,025,625} \approx 4.42067 \times 10^{-9}.
\]