Problem of the Week
Week # 29

When you cut up a figure and reorder the parts, the shape may change but, clearly, the area cannot.

But consider the diagram above. The square is cut into two congruent triangles and two congruent trapezoids. Can we choose \(x\) and \(y\) so the square can be transformed into a rectangle as shown?

A young friend wrote me: “Using graph paper I tried some values of \(x\) and \(y\) but the pieces would not forma a rectangle. When I tried \(x = 5\) and \(y = 3\), the rectangle had an area of \(5 \times 13 = 65\).”

But the square’s area is only 64!

“With a 13 by 13 square \((x = 8, y = 5)\), my rectangle’s area was 168 instead of 169; with a 21 by 21 square \((x = 13, y = 8)\), 442 instead of 441. What is wrong?”

And what part do Fibonacci numbers play in the paradox?