Generally, it is not guaranteed that such a rectangle can be made from a square with same surface area. It is because slope is (slightly) different between each hypotenuse of a triangle and a trapezoid.

It is possible only when slope is equal.

i.e. \[ \frac{x+y}{y} = \frac{x}{x-y}, \] therefore, \( x \) and \( y \) satisfy the ratio of \( \alpha = \frac{1+\sqrt{5}}{2} \).

The answer to another question is that the difference of surface area between the rectangle and square is 1, if \( x \) and \( y \) are adjacent numbers of the Fibonacci series.

The \( n \)th Fibonacci number can be expressed as \( F_n = \frac{1}{\sqrt{5}} \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right\}. \)

So, \( x \) and \( y \) can be expressed as

\[ x = \frac{1}{\sqrt{5}} \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^{k+1} - \left( \frac{1-\sqrt{5}}{2} \right)^{k+1} \right\}, \]

\[ y = \frac{1}{\sqrt{5}} \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^k - \left( \frac{1-\sqrt{5}}{2} \right)^k \right\}, \]

Thus,

\[ x^2 = \frac{1}{5} \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^{2k+2} + \left( \frac{1-\sqrt{5}}{2} \right)^{2k+2} - 2 \times (-1)^{k+1} \right\}, \]

\[ y^2 = \frac{1}{5} \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^{2k} + \left( \frac{1-\sqrt{5}}{2} \right)^{2k} - 2 \times (-1)^{k} \right\}, \]

\[ \alpha y^2 = \frac{1}{5} \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^{2k+1} + \left( \frac{1-\sqrt{5}}{2} \right)^{2k+1} - (-1)^{k+1} \right\}. \]

The difference of surface area is expressed as

\[ (x+y)^2 - \alpha (x^2 + y^2) = -x^2 + xy + y^2 \]

\[ = \left\{ \left( \frac{1+\sqrt{5}}{2} \right)^{2k} + \left( \frac{1-\sqrt{5}}{2} \right)^{2k} - 2 \times (-1)^{k+1} \right\} \times \frac{1}{5} \]

\[ = \left[ (-5)^k \times (-1)^k + \left( \frac{1+\sqrt{5}}{2} \right)^{2k} \right] \times \frac{1}{5} \]

\[ = \left[ (-5)^k \times (-1)^k + \left( \frac{1+\sqrt{5}}{2} \right)^{2k} \times 0 \right] \times \frac{1}{5} \]

\[ = (-1)^{k+1} \]

Here, \((-1)^{k+1}\) is either 1 or -1, dependent on whether \( k \) is odd or even. But in both cases, difference is 1.

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