Equitable Apportionment of Railcars within a Pooling Agreement for Shipping Automobiles

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INFORMS Annual Conference
November 8, 2010

Acknowledgements:
Dr. Moncef Sellami and TTX Reload Group
Omar Nelson Bradley Research Fellowship in Mathematics
Railcar Allocation – Problem Statement

**Given:** The total fleet size to be jointly utilized over the next year by the consortium of automobile manufacturers and railroads participating in the pooling agreement, as determined by a strategic planning (i.e., fleet sizing) process that considers static and dynamic demand profiles.

**Find:** An equitable allocation of railcars to each automobile manufacturer (shipper) from the given total fleet size, with a subsequent equitable allocation of railcar acquisition responsibility for each railroad.
Railcar Allocation Problem:
Fleet Sizing Background

• Spectrum of approaches
  – Reverse routing on OD pairs
  – Solve transportation problem for each shipper and carrier to meet demand ➔ possible reduction in mileage (time) and fleet size
  – Pooled resources across automobile shippers and carriers ➔ greater reduction in time and fleet size

• Deregulation
  – Motor Carrier Act of 1980 (Public Law 96-296)
  – Railroad Transportation Policy Act of 1979 (Public Law 96-448),

• TTX Reload Group established in 1982
  – Coordinates operations of 9 shippers, 17 carriers, and 59,000 auto railcars
  – Saves over 1B miles/year of empty railcar movement over reverse routing
  – Employs static and dynamic fleet sizing, followed by simple shipper and carrier allocation schemes, enabling solutions within five minutes
Railcar Allocation Problem: Fleet Sizing Process

- **Static Fleet Sizing** (Avg. the peak 3 monthly forecasted volumes)

  \[ F_S = \sum_{(i,j) \in OD} \sum_{(p,q) \in A} t_{ij}Q_{ij} + \sum_{(p,q) \in A} \tau_{pq} \bar{x}_{pq} \]
  
  (Loaded railcar days) (Empty railcar days)

  \(Q_{ij}\) values are adjusted based on historical forecast accuracy factors for each shipper.

- **Dynamic Fleet Sizing** (Avg. the 3 highest monthly sizings)

  \[ F_D \Rightarrow DF = F_D / F_S \]

  \[ FS = \sum_{(i,j) \in OD} FS(i, j) = \sum_{(i,j) \in OD} (DF)Q_{ij}(t_{ij} + \phi_i + t_{ij}^q)\beta \]

  (Adjusted for dynamic factor, queuing, bad order factor)

  \[ \phi_i = \frac{\sum \tau_{pi} \bar{x}_{pi}}{\sum_{p:(p,i) \in A} \bar{x}_{pi}} \]

  (Weighted average empty return time to origin \(i\)).
Current Railcar Allocation Scheme

- **Shipper Allocations** \((k \in K)\)
  
  (in proportion to the weighted average empty return time to the origin(s))

  \[
  FS^{PAS}(k) = \sum_{(i,j) \in OD_k} \sum_{i,j} FS(i, j), \ \forall \ k \in K
  \]

- **Carrier Allocations** \((r \in R)\)
  
  (in proportion to the loaded railcar-days of business conducted with each railroad)

  
  \[
  FS_r^{PAS} = \sum_{k \in K} FS_{rk}^{PAS}, \ \forall \ r \in R
  \]

  \[
  FS_{rk}^{PAS} = FS^{PAS}(k) \left( \frac{L_{rk}}{\sum_{\rho \in R} L_{\rho k}} \right), \ \forall \ r \in R, \ k \in K
  \]

  \[
  L_{rk} = \sum_{(i,j) \in OD_k} \sum_{i,j} f_{ijr} Q_{ij} t_{ij}, \ \forall \ r \in R, \ k \in K
  \]

  (fractional responsibility)

  (proportional weighting)
Railcar Allocation Problem: Motivation (1 of 2)

- Neglects **separability of network components** into disjoint subproblems
  - Allows characteristics of a component to inequitably affect allocations for others
  - Each component should be addressed separately

**Legend:**
- : Active loaded flows,
- : Active empty flows.

**Carrier #1**
- Empty & loaded flows

**Carrier #2**
- Empty & loaded flows

**Shipper #1**
- Loaded flows

**Shipper #2**
- Loaded flows

**Carrier Allocations**
- \( FS^{PAS}_{11} = 15 \)
- \( FS^{PAS}_{21} = 15 \)
- \( FS^{PAS}_{12} = 0 \)
- \( FS^{PAS}_{22} = 10 \)

**Carrier-to-shipper Allocations**
- \( FS^{PAS}_{1} = 15 \)
- \( FS^{PAS}_{2} = 25 \)

**Loaded carrier-shipper railcar days**
- \( L_{11} = 5 \)
- \( L_{21} = 5 \)
- \( L_{12} = 0 \)
- \( L_{22} = 5 \)

**Fleet size for OD-Pairs**
- \( FS(1,4) = 10 \)
- \( FS(2,5) = 20 \)
- \( FS(3,6) = 10 \)

**Shipper Allocations**
- \( FS^{PAS}(1) = 30 \)
- \( FS^{PAS}(2) = 10 \)
• If alternative optimal solutions for empty railcar flow exist
  – Shipper allocation via average nodal empty transit times may affect different (i.e., inequitable) shipper allocations and subsequent carrier allocations
  – Must utilize allocation method that is independent of alternative optimal solutions for empty railcar flow

<table>
<thead>
<tr>
<th>(i,j)</th>
<th>(1,3)</th>
<th>(1,4)</th>
<th>(2,3)</th>
<th>(2,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_{ij}</td>
<td>15</td>
<td>10</td>
<td>--</td>
<td>12</td>
</tr>
<tr>
<td>t_{ij}</td>
<td>1</td>
<td>2</td>
<td>--</td>
<td>1</td>
</tr>
<tr>
<td>(f_{i1},f_{j2})</td>
<td>(1,0)</td>
<td>(0,1)</td>
<td>(--,--)</td>
<td>(0.5,0.5)</td>
</tr>
<tr>
<td>\beta</td>
<td>1 2 1 2</td>
<td></td>
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</tbody>
</table>

\phi_1=1.4 \quad FS=106
\phi_2=2 \quad \text{FS}^{\text{PAS}(1)}=70
FS(1,3)=36 \quad \text{FS}^{\text{PAS}(2)}=36
FS(1,4)=34 \quad \text{FS}^{\text{PAS}_1}=48
FS(2,4)=36 \quad \text{FS}^{\text{PAS}_2}=58

\phi_1=1.88 \quad FS=106
\phi_2=1 \quad \text{FS}^{\text{PAS}(1)}=82
FS(1,3)=43.2 \quad \text{FS}^{\text{PAS}(2)}=24
FS(1,4)=38.8 \quad \text{FS}^{\text{PAS}_1}=47.1
FS(2,4)=24 \quad \text{FS}^{\text{PAS}_2}=58.9
### Alternative Allocation Schemes Examined

<table>
<thead>
<tr>
<th>Schemes</th>
<th>To Shippers</th>
<th>Governing Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current</strong></td>
<td><strong>PAS - Loaded Transit Times</strong></td>
<td>$FS^{PAS}(k) = \sum_{(i,j)\in OD_k} FS(i, j), \forall k \in K$</td>
</tr>
<tr>
<td><strong>Proposed</strong></td>
<td><strong>SA1 - Loaded Transit Plus Queuing Times</strong></td>
<td>$FS^{SA1}(k) = FS \left[ \frac{L(k)}{\sum_{k'\in K} L(k')} \right], \text{ where } L(k) = \sum_{(i,j)\in OD_k} Q_{ij}(t_{ij} + r^q_{ij})$</td>
</tr>
<tr>
<td></td>
<td><strong>SA2 - Marginal Cost Analysis #1</strong></td>
<td>$FS^{SA2}(k) = FS \left[ \frac{L'(k)}{\sum_{k'\in K} L'(k')} \right], \text{ where } L'(k) = \sum_{(i,j)\in OD_k} Q_{ij}(t_{ij} + r^q_{ij} + \left( \frac{\psi_k}{\sum_{k'\in K} \psi_k'} \right) - \bar{\nu})$</td>
</tr>
<tr>
<td></td>
<td><strong>SA3 - Marginal Cost Analysis #2</strong></td>
<td>$FS^{SA3}(k) = FS \left[ \frac{L''(k)}{\sum_{k'\in K} L''(k')} \right], \text{ where } L''(k) = \sum_{(i,j)\in OD_k} Q_{ij}(t_{ij} + \phi_i^* + t^q_{ij})$</td>
</tr>
<tr>
<td></td>
<td><strong>SA4 - Shapley Value Allocations</strong></td>
<td>$FS^{SA4}(k) = \sum_{S \subseteq K, k \in S} \frac{(</td>
</tr>
</tbody>
</table>

*Marginal cost examined for each shipper as an entity (SA2), or via its node-wise contributions (SA3)
## Alternative Shipper Allocation Schemes Examined

- **Proposition**: Present carrier allocation scheme (RA1) is equivalent to a game-theoretical approach utilizing Shapley value allocations.
- Cost-based carrier allocations (RA2) are independent of shipper allocations, and based on \( f_r \) —values that are directly proportional to the respective loaded railcar days of business.

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<tr>
<td>Current</td>
<td>RA1 - Loaded railcar movements</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( FS_r^{PAS} = \sum_{k \in K} FS_{rk}^{PAS}, \forall r \in R )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( FS_{rk}^{PAS} = FS^{PAS}(k) \left( \frac{L_{rk}}{\sum_{\rho \in R} L_{rk}} \right), \forall r \in R, k \in K )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( L_{rk} = \sum_{(i,j) \in OD_k} f^k_{ijr} Q_{ijr} t_{ij}, \forall r \in R, k \in K )</td>
</tr>
<tr>
<td>Proposed</td>
<td>RA2 - Total Capital Plus Operating Costs</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Minimize ( \left{ \sum_{r \in R} (f_r - \bar{f}<em>r)^2 : \sum</em>{r \in R} f_r = FS; f_r \geq 0, \forall r \in R \right} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \bar{f}<em>r = \left( \frac{\sigma \sum</em>{k \in K} L_{rk} - \xi_r}{c} \right), \forall r \in R, \text{ where } \sigma = \left( \frac{c(FS) + \sum_{r \in R} \xi_r}{\sum_{r \in R} \sum_{k \in K} L_{rk}} \right) )</td>
</tr>
</tbody>
</table>

- (railcar cost per annum)
- (empty railcar movement cost per annum)
Allocation Schemes – Coding and Implementation

• Considered 10 combinations of current and proposed methodologies

• Coded in C++ to call CPLEX 11.1 using Concert Technology to solve the initial fleet sizing problem, and the additional $K$, $m$, or $2^K-1$ related problems, as appropriate

• Generated 20 test instances based on parametric data from the TTX Reload Group

• Initial results (one instance) for method combinations
  – 20 shippers, 7 carriers, 60 origin nodes, 86 destination nodes
  – Significant variability among shipper allocations
    $s \in [1.98, 49.51]$ for $\bar{x} \in [294.1, 2339.2]$
  – Lesser variability among carrier allocations
    $s \in [0.80, 11.95]$ for $\bar{x} \in [2008.1, 3423.9]$
Relative Changes from Benchmark Allocation Methods (PAS and PAS-RA1) over 30 Instances

- Relative (%) Changes to **Shipper Allocations** from PAS

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<tbody>
<tr>
<td>SA1</td>
<td>2.459</td>
<td>2.997</td>
<td>5.209</td>
<td>-4.805</td>
<td>2.519</td>
<td>-2.444</td>
<td>1.119</td>
</tr>
<tr>
<td>SA2</td>
<td>2.351</td>
<td>2.880</td>
<td>4.904</td>
<td>-4.737</td>
<td>2.424</td>
<td>-2.362</td>
<td>1.142</td>
</tr>
<tr>
<td>SA3</td>
<td>2.349</td>
<td>2.884</td>
<td>4.917</td>
<td>-4.763</td>
<td>2.399</td>
<td>-2.392</td>
<td>1.164</td>
</tr>
<tr>
<td>SA4</td>
<td><strong>1.455</strong></td>
<td><strong>1.891</strong></td>
<td><strong>4.564</strong></td>
<td>-2.378</td>
<td>1.754</td>
<td>-1.095</td>
<td><strong>1.389</strong></td>
</tr>
</tbody>
</table>

- Relative (%) Changes to **Carrier Allocations** from PAS-RA1

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</thead>
<tbody>
<tr>
<td>SA1-RA1</td>
<td>0.0179</td>
<td>0.024</td>
<td>0.035</td>
<td>-0.036</td>
<td>0.019</td>
<td>-0.019</td>
<td>1.074</td>
</tr>
<tr>
<td>SA2-RA1</td>
<td>0.0176</td>
<td>0.023</td>
<td>0.033</td>
<td>-0.036</td>
<td>0.019</td>
<td>-0.018</td>
<td>1.011</td>
</tr>
<tr>
<td>SA3-RA1</td>
<td>0.0173</td>
<td>0.023</td>
<td>0.033</td>
<td>-0.036</td>
<td>0.018</td>
<td>-0.018</td>
<td>1.016</td>
</tr>
<tr>
<td>SA4-RA1</td>
<td><strong>0.0114</strong></td>
<td><strong>0.015</strong></td>
<td><strong>0.023</strong></td>
<td>-0.023</td>
<td>0.012</td>
<td>-0.013</td>
<td><strong>1.225</strong></td>
</tr>
<tr>
<td>PAS-RA2, Sα-RA2, α=1,...,4</td>
<td><strong>0.5655</strong></td>
<td><strong>0.703</strong></td>
<td><strong>0.625</strong></td>
<td>-1.206</td>
<td>0.471</td>
<td>-0.655</td>
<td><strong>0.859</strong></td>
</tr>
</tbody>
</table>

- Testing conclusions
  - Greater variability among shippers’ alternative allocations than those for carriers
  - Inc:Dec ratios provide empirical evidence that a redistribution of capital is in order
  - Even SA4 was not too computationally expensive, requiring an average of 41.0 CPU minutes to solve the instances sized similar to current environment.
Conclusions and Recommendations

• Results
  – Demonstrated need to alter existing aggregate approach and replace current shipper allocation method to prevent inequities
  – Reinforced value of current carrier allocation method
  – Demonstrated several philosophically equitable alternatives

• Future Research
  – Increase granularity of transportation costs with respect to specific cargoes


