Modeling Cooperation in Networks, Organizations, and Systems

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Outline

1. Introduction to Subset Team Games
   - Basic Idea
   - Examples
   - Definitions

2. A Hockey Example
   - Motivation
   - Start simple
   - Some Limitations and Remedies

3. Conclusion
   - Future Work
   - Summary
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Cooperation in Subset Team Games

- A game where players are trying to accomplish some common goal.
- Each player has some positive contribution toward the goal.
- We classify the contributions of a player or subset of players as competitive/altruistic (selfish/unselfish, greedy/not greedy).
- Try to understand cooperation within organizations and teams
  - What group of players yield best results?
  - All unselfish players?
  - All selfish players?
  - Some combination?
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Graph Theory, Networks
(last talk by Elisha Peterson)
Example
Cooperation in Pursuit and Evasion Games.

Brian Macdonald, Chris Arney, Elisha Peterson

Cooperation in Organizations
Cooperation in Pursuit and Evasion Games.
Cooperation in Pursuit and Evasion Games.
1. Can the distribution of teams’ altruistic and competitive contributions tell us anything about the performance of the team?

2. Can a player’s altruistic and competitive contributions be an indicator of future performance? (New circumstances, new teammates, new team)

3. Do certain pairs of players seem to work best? 3-man units? 5-man units?

These questions could be asked about any organization.
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Some Definitions.

$T$ A team of players
$A$ A subset of players
$A^c$ Complement of $A$, $T \setminus A$. All players not in $A$.
$u$ Value function. Assigns value to each outcome.
$u_A(B)$ Represents the value to $A$ when $B$ participates.
Example of Value

Pursuit and Evasion Games.

Value: # of wildebeasts caught.
Value: amount of information transmitted.
\[ u_T(T) \] Value to team, when everyone participates

\[ u_{Ac}(T) \] Value to everyone but A, when everyone participates.

\[ u_{Ac}(A^c) \] Value to everyone but A, when A doesn’t participate.

The competitive contribution of A is

\[ c(A) = u_T(T) - u_{Ac}(T) \]

high when contributions of A are valuable to A.
\[ u_T(T) \] Value to team, when everyone participates
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\( u_T(T) \) Value to team, when everyone participates

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The altruistic contribution of \( A \) is

\[
a(A) = u_{Ac}(T) - u_{Ac}(A^c).
\]

high when the contributions of \( A \) are valuable to everyone else.

The marginal contribution is

\[
m(A) = c(A) + a(A) = u_T(T) - u_{Ac}(A^c).
\]
$u_T(T)$ Value to team, when everyone participates

$u_{Ac}(T)$ Value to everyone but $A$, when everyone participates.

$u_{Ac}(Ac)$ Value to everyone but $A$, when $A$ doesn’t participate.

The altruistic contribution of $A$ is

$$a(A) = u_{Ac}(T) - u_{Ac}(Ac).$$

high when the contributions of $A$ are valuable to everyone else.

The marginal contribution is

$$m(A) = c(A) + a(A) = u_T(T) - u_{Ac}(Ac).$$
Example of Value
Pursuit and Evasion Games.

Value: # of wildebeasts caught. Suppose $A$ is a single lion. $c(A)$ measures # of WB caught by $A$. $a(A)$ measures how well do the other lions do when $A$ is playing vs not playing.
Introduction to Subset Team Games

Basic Idea
Examples
Definitions

A Hockey Example

Motivation
Start simple
Some Limitations and Remedies

Conclusion

Future Work
Summary
Cooperation in Competitive Sports Teams

Sports data is nice because:

1. The data is accurate, complete, and detailed. For hockey:
   - Who is on the ice at every second of every game.
   - All events are recorded (goal, shot, hit, giveaway, etc.).

2. Longitudinal data.

3. Easily quantifiable outcomes and "values" (goals/points scored, wins, etc.). Not subjective.

4. Gain new insights based on stuff learned from this data.

5. Interesting, fun, lends itself easily to student projects

Downside: can’t test all combinations of types of players. Coaches decide who plays together.
Cooperation in Competitive Sports Teams

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Definition of value: The value of a season to a subset of players to be the goals scored by those players during that season.

Some notation:
- \( G = \) goals scored by team \( T \).
- \( g = \) goals scored by player \( A \).
- \( gf = \) goals scored while player \( A \) is on the ice.
Definitions using Goals

Definition of value: The value of a season to a subset of players to be the goals scored by those players during that season.

Some notation:

- $G =$ goals scored by team $T$.
- $g =$ goals scored by player $A$.
- $gf =$ goals scored while player $A$ is on the ice.
More Definitions

- \( u_T(T) \) = value to (goals scored by) the team when everyone participates = \( G \)
- \( u_{Ac}(T) \) = goals scored by A’s teammates (excluding A’s goals) = \( G - g \).
- \( u_{Ac}(Ac) \) = goals scored by A’s teammates, during only the times when A does not play = \( G - gf \).
More Definitions
Competitive Contribution, $c(A)$

- $c(A) = u_T(T) - u_{A^c}(T)$
  (The difference in goals scored by the team, with and without $A$ contributions.)
- $c(A) = G - (G - g) = g$, which is goals scored by $A$. 
More Definitions
Altruistic Contribution, $a(A)$

- $a(A) = u_{A^c}(T) - u_{A^c}(A^c)$
  (The difference in goals scored by $A$’s teammates when $A$ is and is not on the ice.)
- $a(A) = (G - g) - (G - gf) = gf - g$

High when
- $A$’s teammates score a lot when $A$ plays,
- $A$ doesn’t score much.

Team does well when $A$ plays, but $A$ doesn’t get the credit.
## Results

### Top 5 Forwards in Altruistic Contribution

<table>
<thead>
<tr>
<th>Player</th>
<th>Pos</th>
<th>Team</th>
<th>$u_T(T)$</th>
<th>$u_A^c(T)$</th>
<th>$u_A^c(A^c)$</th>
<th>$c(A)$</th>
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<tr>
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**Observations:**

- Players with high $a(A)$ tend to have high assists.
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Observation

\[ a(A) \text{ and Assists are highly correlated} \]

Correlation = .91

Notes:
- Correlation is misleading.
- \( a(A) \) and assists both depend on playing time.
- Not much new info
Observation

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$c(A)$ and $a(A)$ are very position dependent

Let’s color forwards and defensemen differently
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D score less goals
D play more minutes
Lower $c(A)$, higher $a(A)$
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Lower $c(A)$, higher $a(A)$
Another Observation

Team distribution of $c(A)$ and $a(A)$ is less useful than we originally hoped.

![Graph showing altruistic and competitive contributions for top 15 and bottom 15 teams with averages highlighted.]
Another Observation

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Limitation

Team distribution of $c(A)$ and $a(A)$ is less useful than we originally hoped.

\[
\sum_j c(A_j) = G \\
\sum_j a(A_j) = 4G
\]

For each goal, one player has a competitive contribution and the 4 other players have altruistic contributions.
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For each goal, one player has a competitive contribution and the 4 other players have altruistic contributions.
Team Distribution

The distribution of $c(A)$ and $a(A)$ for

- pairs of players, or

- 5-man units

(as opposed to full teams) will hopefully be useful.

Also, fortunately the current $c(A)$ and $a(A)$ will still be useful for players.
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Limitation: \(c(A)\) and \(a(A)\) are playing time dependent

- Correlations were a little misleading. More playing time → more of everything.

- Ideally, playing time (which is out of the player’s control) shouldn’t matter at all.

Remedy: Use Goals per 60 minutes instead of Goals.
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Limitation: Playing time dependent
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Before: $u_T(T) = G$ (team goals)

Now: $u_T(T) = \frac{G}{t_{team}}$ (team goals normalized by playing time)

Or, weighted average of players’ Goals per 60 minutes

$$u_T(T) = \frac{\sum g_i t_i}{\sum t_i}.$$  

The new versions of $u_{Ac}(T)$ and $u_{Ac}(A^c)$ are similar.
Remedy
Use Goals per 60 minutes

Limitation: Playing time dependent
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Or, weighted average of players’ Goals per 60 minutes

\[
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The new versions of \( u_{A^c}(T) \) and \( u_{A^c}(A^c) \) are similar.
Remedy
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Limitation: Playing time dependent
Remedy: Use Goals per 60 minutes instead of Goals.

Before: \( u_T(T) = G \) (team goals)

Now: \( u_T(T) = \frac{G}{t_{team}} \) (team goals normalized by playing time)

Or, weighted average of players’ Goals per 60 minutes

\[
\begin{align*}
    u_T(T) &= \frac{\sum g_i t_i}{\sum t_i}.
\end{align*}
\]

The new versions of \( u_{Ac}(T) \) and \( u_{Ac}(A^c) \) are similar.
Remedy

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The new versions of \( u_{Ac}(T) \) and \( u_{Ac}(A^c) \) are similar.
Cooperation Space
$c(A) \text{ vs } a(A)$ for forwards and defensemen
Correlations
Assists and \( a(A) \) for forwards

\[ \begin{array}{c}
0.5 & 1.0 & 1.5 & 2.0 \\
-0.5 & -0.4 & -0.3 & -0.2 & -0.1 & 0.0 & 0.1 & 0.2 \\
\end{array} \]

Altruistic Contribution vs Assists

\( a(A) \) and Assists are still correlated, but not as much.
Correlations
Assists and $a(A)$ for forwards

$a(A)$ and Assists are still correlated, but not as much.
Correlations

Correlation: 0.06

This seems promising.

Marginal contribution has been decomposed into two uncorrelated components.
Correlations

$c(A)$ and $a(A)$ for forwards

Correlation: 0.06

This seems promising.

Marginal contribution has been decomposed into two uncorrelated components.
Another Limitation

c(A), a(A) depend highly on teammates that player A never plays with.

\[
\begin{array}{cccccc}
\text{LW} & \text{C} & \text{RW} & \text{D} & \text{D} \\
\text{Player A} & \text{Above} & \text{Above} & \text{Above} & \text{Above} \\
\text{Below} & \text{Below} & \text{Below} & \text{Below} & \text{Below} \\
\text{Below} & \text{Below} & \text{Below} & \text{Below} & \text{Below} \\
\text{Below} & \text{Below} & \text{Below} & \text{Below} & \text{Below} \\
\end{array}
\]

Teammates score a lot when A plays, \(u_{Ac}(T)\) is high.
Teammates score little when A doesn’t play, \(u_{Ac}(A^c)\) is low.
\[
a(A) = u_{Ac}(T) - u_{Ac}(A^c)
\]
is high, no matter what A does.
Another Limitation

c(\textit{A}), a(\textit{A}) depend highly on teammates that player \textit{A} never plays with.

\begin{center}
\begin{tabular}{lcccc}
\textbf{LW} & \textbf{C} & \textbf{RW} & \textbf{D} & \textbf{D} \\
\hline
\textbf{Player A} & Above & Above & Above & Above \\
Below & Below & Below & Below & Below \\
Below & Below & Below & Below & Below \\
Below & Below & Below & Below & Below \\
\hline
\end{tabular}
\end{center}

Teammates score a lot when \textit{A} plays, \( u_{A^c}(T) \) is high.
Teammates score little when \textit{A} doesn’t play, \( u_{A^c}(A^c) \) is low.
\( a(\textit{A}) = u_{A^c}(T) - u_{A^c}(A^c) \) is high, no matter what \textit{A} does.
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<tbody>
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<td>Player A</td>
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Teammates score little when A doesn’t play, $u_{Ac}(A^c)$ is low.

$a(A) = u_{Ac}(T) − u_{Ac}(A^c)$ is high, no matter what A does.
Another Limitation

c(A), a(A) depend highly on teammates that player A never plays with.

<table>
<thead>
<tr>
<th>LW</th>
<th>C</th>
<th>RW</th>
<th>D</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Player A</td>
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Teammates score little when A doesn’t play, \( u_{A^c}(A^c) \) is low.

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\text{LW} & \text{C} & \text{RW} & \text{D} & \text{D} \\
\hline
\text{Player A} & \text{Above} & \text{Above} & \text{Above} & \text{Above} \\
\text{Below} & \text{Below} & \text{Below} & \text{Below} & \text{Below} \\
\text{Below} & \text{Below} & \text{Below} & \text{Below} & \text{Below} \\
\end{array}
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Teammates score a lot when A plays, \( u_{Ac}(T) \) is high. Teammates score little when A doesn’t play, \( u_{Ac}(A^c) \) is low. \( a(A) = u_{Ac}(T) - u_{Ac}(A^c) \) is high, no matter what A does.
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c(A), a(A) depend highly on teammates that player A never plays with.

Could arise in pursuit and evasion games, communication networks, etc.

Two “players” are part of the same team and play the same game, but never play together or never interact in any way.
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Two “players” are part of the same team and play the same game, but never play together or never interact in any way.
Recall $u_T(T) =$ weighted average of each player’s goals per 60 minutes:

$$u_T(T) = \frac{\sum g_it_i}{\sum t_i}$$

Remedy: When computing $u_T(T)$, etc., weight calculations by playing time with player A, instead of total playing time.

Result: Teammates who never actually play with A do not affect $c(A)$, $a(A)$, etc.
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Result: Teammates who never actually play with A do not affect \( c(A) \), \( a(A) \), etc.
Remedy

Weight by playing time with player A

Before: \( u_T(T) = \frac{\sum \frac{g_i}{t_i} t_i}{\sum t_i} \)

Now: \( = \frac{\sum \frac{g_i}{t_i} w_i}{\sum w_i} \)

where \( w_i \) are playing time with A.

If a player \( i \) never played with A, then \( w_i = 0 \), and \( \frac{g_i}{t_i} \) doesn’t factor into the computation.
Remedy
Weight by playing time with player $A$

Before: $u_T(T) = \frac{\sum g_i t_i}{\sum t_i}$

Now: $= \frac{\sum g_i w_i}{\sum w_i}$

where $w_i$ are playing time with $A$.

If a player $i$ never played with $A$, then $w_i = 0$, and $\frac{g_i}{t_i}$ doesn’t factor into the computation.
Unlike the subset team games, when you “remove A" from the game, you don’t play with one less player.

Rather, A is replaced by someone else.

This idea arises in other situations, like in organizations, networks, pursuit and evasion games.

- Sometimes remove a player.
- Sometimes replace a player.
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- Sometimes replace a player.
Remedy

Use replacement players

Define the meaning of a replacement player (ex. playing time <100 minutes).

When calculating $u_{Ac}(T)$, etc, remove $A$’s contribution, but also add the replacement player’s contributions.
Remedy
Use replacement players

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When calculating $u_{Ac}(T)$, etc, remove A’s contribution, but also add the replacement player’s contributions.
Summary of Remedies

- Use Goals per 60 minutes
- Weight based on playing time with A
- Replace A with “replacement player”
After applying all these remedies:

Pretty Picture

$c(A)$ vs $a(A)$ after applying all remedies

Forwards
Defensemen
Average F
Average D
After applying all these remedies:

**Correlation: 0.10**
We’ve used only goals. They are other ways to contribute. Remedy 1: Use Goals For instead of Goals.
Limitation
Used only Goals

We’ve used only goals. They are other ways to contribute.
Remedy 1: Use Goals For instead of Goals.
Remedy 2: Estimate how different box score stats contribute to team success, and use those stats.

Example: Use shots.

- Shots are a better than goals at predicting future goals.
- More data
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Outline

1. Introduction to Subset Team Games
   - Basic Idea
   - Examples
   - Definitions

2. A Hockey Example
   - Motivation
   - Start simple
   - Some Limitations and Remedies

3. Conclusion
   - Future Work
   - Summary
Ideas for Future Work

Pairs, 5-man units, etc.

Find competitive and altruistic contributions for pairs (ex. find $a(A_1 \cup A_2)$). Ditto for 3-man lines, and 5-man units.

- What kinds of pairs of players play best together?
- Does team distribution become useful?
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Analyze contributions in Pursuit and Evasion games

- Possibly more instructive to start with these games, then move to real data.
- Can control team distribution of $a(A)$ and $c(A)$. What happens with a team of pursuers that are all unselfish?
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2. Examples
3. Definitions

A Hockey Example

4. Motivation
5. Start simple
6. Some Limitations and Remedies

Conclusion

7. Future Work
8. Summary
Summary

- Decomposed player contributions into $c(A)$ and $a(A)$.
- Noted several limitations and remedies
- Team distribution of $c(A)$ and $a(A)$ does not appear helpful (with current definitions of value)
- $a(A)$ and $c(A)$ are uncorrelated.