Emergency Deployment to Panama

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Introduction

Your battalion has recently been deployed to Panama in response to an immigration crisis. Because you know you will be there for several months as a response force in the event of an uprising, your theater commander has authorized funds for the construction of a base camp. As the assistant battalion S3, you are charged with several missions which involve the preparation of the camp for your battalion. Most of these missions involve mathematical modeling.

Example 1: You need to construct a gray water pit for the battalion. A gray water pit is a large rectangular hole where water from showers collect so that the ground can absorb the water. You must construct an open top rectangular pit with a square base of x feet on each side and a height of y feet. The pit will be placed so that it can catch all of the runoff water from the showers. It must be large enough to support the peak shower time (right after PT) which is for one hour. Twenty shower heads usually run during this hour, each releasing about 2 cu. ft per minute. Consider the absorption rate of the ground to be negligible.

The costs associated with the construction of the pit involve not only the material, but also an excavation charge. The total cost can be written as $c(x,y) = 5(x^2 + 4xy) + 10xy$. Since the battalion must contract with the local Panamanian engineers for this project, your commander has directed you to find the dimensions of the pit that will minimize the cost. What are

these dimensions and what is
the cost?

Assumptions
We must assume the following: the absorption of the water by the ground will be negligible; all of the water from the shower heads will end up in the pit; the shower heads run continuously and that the rate of each is exactly 2 cu. ft. per minute; and the given cost function is an accurate representation of the associated costs.

Variables:
x : the length and width of the bottom of the pit in feet
y: the height of the pit in feet
c(x,y): the total cost of the construction of the pit in dollars

Since the base of the pit will be a square, we know that the volume will be $v = x^2y$. This volume must be able to support the amount of water coming from the showers. Thus,

$$V = x^2 y = 20(2)\frac{ft^3}{\min} \times 60\min = 2400$$
$$y = \frac{2400}{x^2}$$

Now we can substitute this in for y in the cost equation:

$$c(x, y) = 5x^2 + 20xy + 10xy = 5x^2 + 30xy$$
$$c(x) = 5x^2 + 30x(\frac{2400}{x^2}) = 5x^2 + \frac{7200}{x}$$

Differentiating c(x) gives us:

$$\frac{dc(x)}{dx} = 10x - \frac{7200}{x^2} \Rightarrow x = 19.31$$

We take the derivative above of the cost function and set it equal to zero because we are trying to minimize cost. We then can find y and ultimately c(x,y) for the given critical value of x.

$$x = 19.31\,ft$$
$$y = \frac{2400}{19.31^2} = 6437\,ft$$
$$c(x, y) = 5(19.31)^2 + 30(19.31)(6.437) = \$5593.33$$

Example 2: From research you have found that without restriction a typical battalion consumes electricity in a field environment according to the function $f(t)=\frac{t^4}{4}-t^3+t^2-\frac{1}{6}$, where *t* is in 6 hour increments and f(t) is in KW. (A negative f(t) represents zero KW.) At midnight t = 0, and at 0600, t = 1. The local electrical company can provide a maximum of 2KW at any given time to your location because of the poor condition of the electrical lines.

Must you recommend to your commander a policy on imposing light restrictions between the hours of 0400, when battalion activity begins and 2100? If you determine that you must impose the restrictions, when must you impose them?

Assumptions

We must assume that the model f(t) accurately reflects our battalion's energy consumption. Finally, we will assume that the negative values for this function have no meaning.

Variables:

t: time in 6 hour increments

f(t): consumption of electricity in KW

If we take the derivative of the function and set it equal to zero, we will find possible extrema. We will then use the second derivative to determine if they are extreme points and what type.

$$f'(t)=t^3-3t^2+2t=0$$

This gives us 3 solutions: t = 0,1,2. Using a second derivative test:

$$f''(t)=3t^2-6t+2$$

$$f''(0)=+(\min)$$

$$f''(1)=-(\max)$$

$$f''(2)=+(\min)$$

Since when t = 1, we have a maximum, we find that f(1) = .083 KW, which is below 2KW.

We must also check the endpoints. They are t = 2/3 (actual time 0400 hours) and t = 3.5 (actual time 2100 hours).

f(2/3) = .0309 KW
f(3.5) = 6.72 KW, yes this exceeds the 2 KW, so we have to impose restrictions.

To find when we must impose the restrictions, we will set the original function equal to the 2 KW to see when it exceeds the capacity.

$f(t) = 2 = \frac{t^4}{4} - t^3 + t^2 - \frac{1}{6}$ This yields 4 roots, 2 of which are complex, one negative, and one when t = 2.985 or an actual time of 1754 hours. Seems about the time when all of the radios are hot with the evening reports that are due to higher headquarters!! So we must impose restriction between 1754 hours and 2100 hours.

Now determine when the usage is increasing and decreasing for the given interval.

We know that when t = 0, 1, 2, the electricity usage is neither increasing nor decreasing because the slope of f(t) is zero. If we pick numbers in the following intervals:

$-\infty < t < 0$

$0 < t < 1$

$1 < t < 2$

$2 < t < \infty$

and then substitute into the derivative of the original function, we find that the first interval yields a negative number, meaning the function is decreasing on the first interval. A similar analysis for each interval yields:

$-\infty < t < 0 \Rightarrow decreases$

$0 < t < 1 \Rightarrow increases$

$1 < t < 2 \Rightarrow decreases$

$2 < t < \infty \Rightarrow increases$

This is how we knew that when t = 2.985, the time when the usage equals 2KW, that it would exceed 2 KW up until the time of interest, 2100 hours, or t = 3.5. The function is increasing in this interval. Final analysis is confirmed by looking at a graph of this function in Figure 1, which shows points and intervals of interest.

$$f(t) := \frac{t^4}{4} - t^3 + t^2 - \frac{1}{6}$$

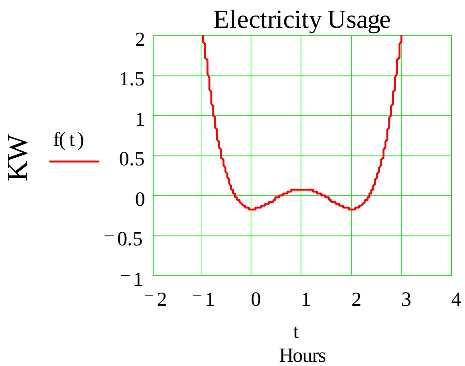

Figure 1

Conclusion

In mathematical modeling we are trying to find a mathematical relationship among variables of a complex situation. Once this relationship is determined, we can analyze a situation in order to predict the results of some course of action[1]. In both of the above problems, we were given mathematical relationships. Our objective was to find optimal points of interest of these relationships or functions. Essentially, we wanted to predict the results of some courses of action. Using the given functions, we were able to find the least expensive dimensions for excavation of a water pit and the times during the day when 2 KW of power were exceeded in the base camp. In both problems we differentiated the functions, set the results equal to zero, and then solved for the corresponding roots in order to find the optimal points, a procedure referred to as optimization. Once a relationship among the variables in a complex situation is found, the situation can usually then be optimized to determine some desired end state. Through optimization, many crucial resources can be saved.

Exercises

1. Because the area around your base camp could potentially be a hostile environment, your commander would like to surround your camp with some sort of protective barrier. You ask around and are able to find 80 rolls of concertina wire. You learn that each roll is 15 meters long, and that a concertina fence is most effective when installed as a triple standard concertina fence (see figure 2 below). Using this concertina as the barrier, what is the maximum amount of area inside the fence you would be able to have for your camp? The location where your commander wants the camp is directly adjacent to a lake. Thus, you do not have to worry about a barricade on that side. Additionally, you can leave

a gap in the fence of 20 meters for incoming and outgoing traffic; a 24 hour guard will be there (see figure 3 below).

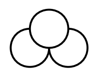

Figure 2 Cross Section

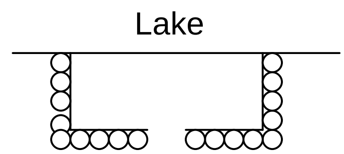

Figure 3 Layout

2. Your home duty station is preparing to send you repair parts for your vehicles. Before they do however, they have to purchase the boxes to send the parts in. Because of your mathematical modeling experience, they ask you for help in determining what size of box they should purchase. The U.S. Postal Service will accept boxes for shipment overseas only if the sum of its length and girth (distance around) does not exceed a certain dimension; in this case 225 inches. You decide to design the box with a square end in order that the bulkier parts can fit. What dimensions will give a box with this design the largest possible volume?

3. One of your responsibilities is to plan and order fuel for all of the vehicles in your unit. Assume the daily consumption of fuel to be best modeled by the equation
$f(t) = 20 + 15 \cos(t)$ where t is in hours and f(t) is in hundreds of gallons (at midnight $t = 0$). What is the maximum amount of fuel that you must plan on ordering from the local community? Approximately what times of the day will these peaks occur? Also, you only have a six man section to run the fuel point. When can you have minimum manning at your fuel station?

4. Now that you have successfully planned for the gray water pit for the battalion, you must now plan for obtaining the water. You are told that the daily consumption of water in a battalion is modeled by $f(t) = t^4 - 18t^2 - 10t + 120$, where t is in hours and f(t) is in tens of gallons of water. When $t = -6$, it is 0600 hours. The local water source plant can meet any demand, however, the piping network that exists near your camp can only provide a maximum of 500 gallons at any given time. One other potential problem is that the local source must provide continuous service. This is because the cost of shutting off and restarting their equipment is expensive. First, verify that the above equation makes sense for your battalion's water consumption. Then, determine if the piping network will meet your needs. If not, what should you do? Finally, will you meet the requirement of the water source plant?

References

1. Giordano, Frank R., and Maurice D. Weir, *A First Course in Mathematical Modeling*, Books / Cole Publishing Company, Monterey, California, 1985.