

# Mathematical Research and Modeling for the Army

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## Introduction

Mathematical sciences have great impact on a wide range of Army needs. The long-term goal of the Army's mathematical research programs is to help the Army develop enhanced capabilities for the 21st century in areas such as materials, systems, testing, evaluation, acquisition, training, and logistics. Mathematics plays an essential role in *modeling systems*, in analyzing and controlling complex phenomena, and in designing and improving systems of critical interest to the Army.

We discuss some of the Army interests that lend themselves to solution through mathematical research developments and the mathematical topics involved in such work. We especially note the role that modeling plays in this effort by highlighting (*through use of italics*) sections where *modeling* is explicitly mentioned and explained. This article is rewritten for the undergraduate audience of this book from a booklet prepared by the Army Research Office [1]. Some of the higher-level mathematical subjects and scientific terminology excerpted from [1] and mentioned in our discussion may be unfamiliar to undergraduates. However, we hope all readers obtain a good view of the magnitude, flavor, scope, and importance of mathematics in the Army. Researchers interested in a more complete and technical review of Army mathematics should refer to [1] and [2], specifically the sections in Chapter IV (Technical Developments) and Chapter V (Basic Research).

## Objectives and Components

With the advent of high performance computing, the mathematical sciences have become an integral part of every scientific and engineering discipline. Computing, in the form of simulations, now complements analysis and experiments as part of a triad that is increasingly successful in understanding physical, biological and behavioral phenomena. Computing and simulation have become essential tools for the Army of the future. Development of intelligent information processing makes possible the digitized battlefield. Real-time acquisition, representation, synthesis, and distribution of vast amounts of battlefield information are key ingredients of the digital battlefield. The USMA Department of Mathematical Sciences is actively engaged in research, consulting, and problem solving in the area of digitization of the battlefield.

The major technical objectives of the Army's mathematical research programs are to develop new mathematical theories, *methods for modeling*, analysis, algorithms, design, and control of physical, biological, and cognitive processes,

and to make possible future intelligent systems through progress in information processing.

The objective of the Army's efforts in basic research is to provide a well-equipped strategic force capable of decisive victory in conflicts in the Information Age. Advances in the areas of Army interest depend, in part, on advances in a number of mathematical disciplines. The six components of Army mathematical research are:

- Applied Analysis
- Computational Mathematics
- Probability and Statistics
- Systems and Control
- Discrete Mathematics
- Intelligent Systems

Many Army systems have long development cycles: five-, ten- or twenty-year horizons from conception to implementation. Accordingly, mathematical research issues that play critical roles in the success of new systems have to be foreseen and addressed up to twenty years in advance. This proactive research stance requires constant interaction between researchers, Army development personnel, and Army field soldiers.

To be a valuable contributor, mathematical research must address realistic issues of potential long-term Army interest. We must accelerate the passage of new ideas from the theoretical stage to the implementation stage. The mathematical sciences cannot just be a problem solving activity conducted before, or in parallel with, developmental work. Mathematical research must be interactive and collaborative with other sciences, engineering, and development activities to ensure that Army systems will be built quickly and will be built right the first time. Continuous technological collaboration is becoming a much more important mode of work than just being a specialist who adds value at one's assigned point of the development cycle and then passes the project on.

### **Army Interests**



The objective of Army basic research is to provide a well-equipped force capable of decisive victory in conflicts in the Information Age. To achieve this objective, advances in the following specific areas of application and *modeling* are needed:

- Advanced materials and materials manufacturing processes (aircraft skin, turbine blades, etc.)

- Behavior of materials under high loads, including failure mechanics (penetration mechanics of sabot into armor, etc.)
- Structures, including flexible structures (next generation vehicles/aircraft, parachutes, etc.)
- Fluid flow, including reactive flow (flows undergoing chemical reactions, such as fuel combustion and propellant burn, etc.)
- Power and directed energy (rechargeable personnel and vehicular sources, directed energy weapons, etc.)
- Microelectronics and photonics
- Sensors (on the battlefield, embedded in equipment, etc)
- Control and optimization, distributed to user/system level
- Information processing
- Interactive simulation, distributed to user/system level (*combat modeling*)
- Design and validation of software and large software systems
- Automatic target recognition
- Intelligent (adaptive/anticipatory) systems; human/system interface
- Battlefield management
- Soldiers and aggregates of soldiers as systems: *behavioral modeling*, performance, mobility, heat-stress reduction, camouflage, chemical and ballistic protection

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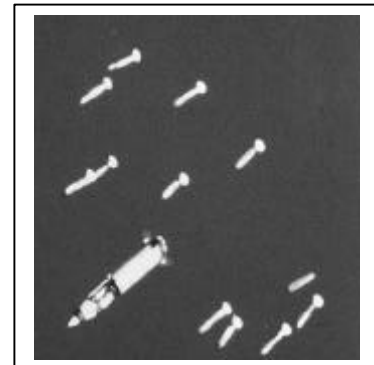
### **Major Components of Army Mathematics**

The six components of Army mathematical research are:

Applied Analysis  
 Computational Mathematics  
 Probability and Statistics  
 Systems and Control  
 Discrete Mathematics  
 Intelligent Systems

We describe some of the activities and modeling issues involved in each of these areas:

**Applied Analysis:** Current interest in applied analysis is in *mathematical modeling of difference equations, differential equations, and integral equations* for advanced materials, fluid flow, and nonlinear dynamics.



Applied Analysis supports mathematical research in advanced materials; the goal is to optimize properties or performance characteristics of new materials, including “smart” materials, advanced composites and biologically inspired materials. Smart materials are the functional ingredients of actuators, sensors, and transducers. Such materials undergo transformation when some mechanical, thermal, electrical, or magnetic factor changes. Lightweight, high-strength structural components, including advanced composites, contribute to mobility and protection requirements for U.S. forces. Advanced composites are challenging to analyze and design because of the presence of many interacting length scales.

Understanding and *modeling fluid flow* is an area in which many of the basic equations are well known but which requires further work because of the presence of vast ranges of length scales and the complexity (nonlinearity) of the phenomena. For *modeling* and optimization of airflow around rotors and of combustion, detonation, and explosion of reactive flows, further research is needed on two-phase flows (such as the dispersion and transport of a liquid agent in a high-speed airflow). This is directly related to Army interests in theater missile defense.

Army needs in the nonlinear dynamics area are in optical, electro-mechanical, and other nonlinear phenomena. Enhanced electro-optic devices and improved device manufacture enable control, guidance, and display for Army systems with embedded electronic computers, such as sensors, smart munitions and display panels. Activation and interrogation of nonlinear sensors is important. Nonlinear wave motion is a central issue in fiber-optic transmission. *Physics-based mathematical models of the human dynamic* may need to be created to model and study the soldier as a system in a variety of environments.



#### **Computational Mathematics:**

The evolution of the Army into a modern, technology-based force places increasing demands on numerical methods and optimization for faster, more stable, and accurate solutions to problems in the physical sciences. The objectives are to develop numerical methods and optimization procedures for *models of physical, biological, and resource-management phenomena* of interest to the

Army and to develop efficient computational algorithms for implementing these numerical methods.

Key application areas are computational fluid dynamics for ballistics and rotorcraft, combustion, detonation, mechanics of penetration, material behavior, computational chemistry, simulation of large mechanical systems, vulnerability analysis, logistics, and resource management.

The areas of investment in numerical methods include methods for efficiently solving nonlinear partial differential equations, methods for shock wave phenomena, methods for solving constrained ordinary and partial differential systems, parallel computing techniques for linear algebra problems arising from *discretizing continuous models*, and parallel computing techniques for calculating eigenvalues arising in the analysis of large structures. Algorithms for penetration mechanics based on *improved physics models* are important. New computational procedures for acceleration-induced mixing are needed for an enhanced understanding of detonation. Simulations of fluid-structure interaction and behavior of flexible structures, including ram-air parachutes, are required and need to be developed.

Integer programming (where some or all variables are required to take on integer values) and nonlinear optimization for large-scale problems are important for C3 applications, vulnerability analysis, logistics, resource management and design of materials. Topics of interest include nonlinearly constrained optimization and optimization methods for singular problems. Parallel computing methods for molecular dynamics with hundreds of millions of particles, needed for the design of new materials, are needed. New, faster algorithms for distributed real-time management of communication networks, where not all of the information is known in advance, is needed for efficient approximation algorithms.

**Probability and Statistics:** Research in probability and statistics supports critical Army needs in decision making under uncertainty. Areas include probabilistic and statistical analysis of *models of physical and operational phenomena* of interest to the Army and development of testing and estimation procedures.

Many Army R&D programs are directed toward system design, development, testing, and evaluation problems that depend on analyzing stochastic (probabilistic) dynamical systems. Such problems generate a need for research in the field of stochastic processes. Special emphasis is needed on research into methods for analyzing observations from phenomena modeled by such stochastic processes and to numerical methods for stochastic differential equations. Research areas of importance to the Army in probability and its applications include interacting particle systems, probabilistic algorithms, control of stochastic processes, large deviations, simulation methodology, spatial processes, and image analysis.

Data collected by Army R&D programs are frequently gathered in nonstandard situations. Problems inherent in the collection of field data, where conditions cannot be tightly controlled, lead to the production of messy data. Current statistical methods perform well in determining information from medium-sized data sets collected under “reasonable” conditions using moderately well understood statistical distributions. However, the Army often has only very large data sets or very small amounts of data sampled from nonstandard, poorly understood distributions.

Large data sets typically occur when computers are used to acquire data. The quality of the data is often varied since not much attention is paid to controlling precision. Quantity is sometimes substituted for quality. The problems to be studied are vague and, oftentimes, have not been formulated before acquiring the data. Methods are being developed *by statisticians and scientists to model and analyze such data*. For real scientific progress, it is important to study the vast array of problems of this type: find elements of commonality in well delineated contexts, bring in appropriate probabilistic structures, develop new probabilistic and statistical theories, discover optimal statistical procedures, exploit the dependencies in the data, and make these methods accessible to the user by developing software to carry out the statistical analysis.

On the other hand, there are situations in testing and on the battlefield where the Army can collect only small amounts of data due to cost, time, and safety constraints. New statistical methodologies and close collaboration with scientists in related fields are required to address the problem of extracting information from meager samples. Understanding of physical processes must be combined with statistical theory. In order to extract more information out of less data, improved methods for combining information from disparate tests are needed.

Statistical procedures and theory are needed to analyze data on repaired items. When projecting life cycles of brand new or newly repaired items, one does not have the luxury of possessing data on the mature or aged item, as is often assumed. *The models of minimal repair* currently in use are not adequate in many instances.

Composite materials, like ceramics, have high tensile strength and can withstand high temperatures but are typically brittle, which leads to catastrophic failures. For these materials, there is a large variability in failure times, so deterministic estimation of reliability is a challenge. One needs both probabilistic methodology that describes the stochastic nature of the growth of cracks in different media and also associated statistical analysis. Integration of statistical procedures with scientific and engineering information about failure mechanisms is important. Investigation of the theory of extreme values with appropriate modifications to account for crack size dependence, uncertainty about the

physical constants, and spatial structure is in order. The analysis of actual data will call for Bayesian methods to incorporate physical knowledge about failure processes.

Spatial point processes motivated by detecting mine fields in the presence of clutter are important for Army operations. More research is required to solve this problem in statistical areas including Bayesian methods, *Markov random models*, cluster analysis, Markov chains, and Monte Carlo methods.

**Systems and Control:** The systems and control effort is *concerned with modeling, analysis, and design of real-time systems*, especially as they relate to Army problems in distributed command, control, and communications and in guidance and control of automated systems. Our goal is to enhance the understanding of control and of controllability of systems, especially real-time systems.

Topics of interest include control in the presence of measurement uncertainties, geometric theory of control, robust and adaptive control for multivariable and nonlinear systems, synthesis of time and frequency domain issues in infinite dimensions, system identification, and computational issues related to the design and implementation of real-time control systems.



Some practical issues of interest in control theory include control for smart structures, robotics, micromachines (such as chip-based accelerometers), wireless communication systems, visual tracking, and parallel computing. In the area of hybrid control, we are interested in research on the use of smart motors in real-time feedback control and on robots that intelligently interact. A control system with minimal complexity must be imbedded into the material. For tele-robotics and micromachines, we are interested in research that improves visualization and produces finer manipulators, new remote sensors, and new ways to integrate everything into a complete system. In hospitals, for example,

more automation is needed in the operating rooms. Tele-operation is relevant to the Army's need to get medical help to the soldier on the battlefield as soon as possible. We are also interested in the control issues of wireless communication systems. Research is needed to develop novel techniques for using visual information in control systems. New research is needed that combines techniques from signal and image processing, computer vision, and control. We need to develop novel means of acquiring, storing, manipulating, and synthesizing signals and images for use in a feedback loop. Army needs are in the areas of remotely controlled weapons and vehicles, manufacturing systems, and automatic target recognition. Research is needed to apply massively parallel computers to the design of robust control systems. This will allow for coverage of a broader range of parameters, operating conditions and design alternatives.

Research in foundations of intelligent control systems (defined more fully below) is the cornerstone for a broad range of Army applications such as automation, robotics, and distributed C3. Research in this area includes understanding discrete event dynamical systems and distributed communication and control. Large intelligent control systems usually have intensive computational requirements. *Research on the mathematical modeling to meet these requirements is a challenging area.* The fundamental concept of a system that can process artificially sensed information, make optimal decisions based on this information and well-defined objectives, and translate those decisions into actions is a guiding and unifying theme for basic research in all major aspects of this area. Such research is important for a control and communication system for a remotely tele-operated vehicle. It is widely recognized that it is technologically difficult and operationally ill advised to build complex systems that cover large geographical areas and are controlled in a centralized way. Distributed control and communication systems have, therefore, become increasingly important. Intelligent control is the avenue by which control systems will be expanded to more general functions of decision making, goal selection, mode switching, assistance to human operators, scenario identification, and system adaptation.

**Discrete Mathematics:** Discrete mathematics research in the Army involves *developing and analyzing mathematical models of discrete phenomena* of interest to the Army and enhancing the applicability of these models by developing usable algorithms.

The key requirements of the digital battlefield include real-time acquisition, representation, reduction, and distribution of vast amounts of battlefield information (logistical, intelligence, medical information, command and control). Discrete mathematics plays a key role in effectively implementing the digital battlefield. This area is also crucial for success of the enabling technology of



advanced distributed simulation used in determining and analyzing alternatives for battlefield effectiveness.

The foci of *discrete mathematical modeling* are developing and analyzing discrete problems in computational geometry, computational algebra, logic, network flows, graph theory, and combinatorics. Computational geometry is a powerful tool for a number of applications such as robotics, autonomous navigation, battle management, C3, and manufacturing. Specific areas of emphasis include robust geometric computation, solid modeling, parallel and distributed computing, and interactive visualization techniques. Other areas of interest include distributed algorithms for network flows, randomization in computing, and symbolic methods.

Symbolic methods have a growing number of Army-related applications in control, robotics, and computer vision. Needs are in computational algebraic geometry techniques for solving polynomial systems, discrete methods for combinatorial optimization, symbolic methods for differential equations, mathematical logic, and formal language theory.

**Intelligent Systems:** Intelligent systems are a new area of mathematical research and are best described as systems that can perform satisfactorily in the presence of uncertainties arising from changes in the environment in which they are situated. In order to do this, the systems must learn about the unknown or partially known environment, track the changes and adapt themselves to the evolving environment possibly by changing their structure and their goals. In other words, intelligent systems exhibit emergent behavior.

Intelligent systems are modular and involve a hierarchy of feedback loops with a multiplicity of actuators and sensors. At the lower end of the hierarchy, the signals involved are physical signals while, as one goes up the hierarchy, one has to sense and manage signals that are more abstract. The flow of information about the physical world to the cognitive world is not one-way, but involves complicated feedback between various levels of the hierarchy.

Work in classical feedback theory has demonstrated that a simple system can achieve satisfactory performance even in the face of relatively poor knowledge of the environment. The methodology of feedback control is widely used today in process control, weapon system design, aerospace systems and many defense systems. The success of the methodology depends on the fact that it is focused on a well-defined class of problems *with well-defined models for dynamics and measurements*. Intelligent systems that are capable of successful autonomous operation in spite of unanticipated structural changes in the environment are becoming more prevalent.

The original promise of artificial intelligence (AI) was that it would provide a flexible framework for the development of intelligent systems capable of extracting information, making decisions, and applying control. Initial work in AI concentrated on universal aspects of this problem with limited success. It soon became clear that the spaces through which one had to search in order to choose optimal actions were vast, much larger than computing resources could handle by exhaustive search techniques. It became important to find heuristics that limited the search to a tractable subspace, which would contain a good choice.

The Army has great need for intelligent systems in

- Distributed command and control
- Simulated battlefield environment
- Intelligence augmentation of human centered systems
- Automatic/aided target recognition
- Smart medics
- Manufacturing systems.

### **Inter-Component Opportunities**

It is important to underscore a few advanced technologies in which progress depends on more than one of the six areas above. One such example is simulation. *Advances in modeling techniques* and interactive computational capabilities are needed to support stochastic modeling and simulation of combat, to assess the impact of changes in doctrine and tactics, and to determine the cost-effectiveness of new systems for the battlefield. Simulation is a key and enabling technology in determining and analyzing alternatives for digitizing the battlefield.



The foundations of Army force-on-force *combat models* and simulations have not been significantly improved since the development of the Lanchester equations. Lanchester equations are principally firepower equations, hence outcomes are dependent on firepower rates. This type of *model and simulation* is not the choice for supporting Army 21<sup>st</sup> century critical requirements. The information age has made a significant difference in the

*complexity of modeling combat information warfare*. The lack of research into better techniques of *modeling military operations* to include the representation of information operations has caused a shortfall in our ability to assess future warfighting concepts, such as Force XXI and Army After Next.

We need *modeling and simulation analytical tools* to provide the underpinnings necessary for many future warfighting concepts. Warfare has changed due to information technology and the importance of cognitive command and control integration of the soldier into future combat environments. The dynamics of the battlefield have changed from firepower and attrition to a more complex interaction of information technologies and smart weapon systems. This has resulted in a multi-scale, dynamic environment. Progress is needed in parameter estimation, aggregation methodology, algorithm development, and system verification to support *distributed predictive models*. The mathematical sciences areas of research required to do this are: *stochastic modeling*, non-linear dynamic representation, computational mathematics, probability and statistics, and software engineering.

Substantial improvements in current simulation technologies are needed to enable professionals at widely distributed sites to interact simultaneously through simulators, simulations, and deployed systems in synthetic operational environments. A fundamental issue for improving simulation is the creation of a technology for *constructive integration of diverse models*. Advanced signal processing is another example of an area in which progress depends on research in the six program areas mentioned above. Advanced signal processing is key to the digitization of the battlefield. Substantial advances in development, design, and implementation of efficient procedures that exploit computing environments require progress in computational methods and in stochastic methods. Finally, fundamental advances in information sciences are needed for winning the information war. *Mathematical models are needed for rapid design, development, and dynamic evolution of software systems.*



## **Conclusion**

There is general agreement that mathematical research is extremely important to the Army. *As the reliability of simulations based on mathematical, statistical, and information-theoretic models increases* and as physical experiments become increasingly costly to carry out, the need for contributions from the mathematical sciences increases. The focus of this discussion has been on the research in mathematical sciences needed to achieve Army goals and support the Army in the field. In these roles, mathematics is a valuable and effective force multiplier for the US Army.

## **Exercises**

1. What is mathematical modeling? Summarize the areas in which modeling is important to the Army and why.
2. Describe the relationship between data, information, models, and mathematics. Describe the value of each of these items to Army Research and Development activities.
3. If Army budget funds become scarcer, speculate on how the ratio of funds allocated to the areas of experiments, computations, and analysis may change. Give the reasons for your predictions.
4. Describe the battlefield of 2025. What kind of systems do you see? Will mathematics play a role in developing those systems?
5. What will be the future threat to America's military? What will be the required capabilities of the Army in the future? Will mathematics play a role in developing those capabilities?

## **References**

[1] Army Research Office (Mathematical and Computer Sciences Division), *Investment Strategy in the Mathematical and Computer Sciences*, 1996.

[2] Department of the Army, *Army Science and Technology Master Plan* (2 volumes), 1997.