Topological Terrain Map Resolution: Intervisibility, Crest, and Military Crest

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Introduction

The depiction of terrain in computerized combat simulation models generally takes the form of a discrete approximation. The modeled terrain surface is often represented by a step function over a two-dimensional domain covered by a tessellation of squares or hexagons. Military field testing applications use such digitized terrain maps in connection with target tracking enhancements. Such uses include improvements in accuracy of tracking in the elevation dimension ("*z*-axis") using Global Positioning System (GPS--satellite) information. Digital maps are used in many modeling applications in connection with computer displays of units on or above the terrain.

In the case where the domain of the approximating terrain surface is covered by squares, it is common to designate the terrain resolution in terms of the length of a side of such a square. Thus, "100-meter terrain", refers to an approximating function with domain partitioned into squares 100 meters on a side. As the resolution is increased (25-meter and 3-meter terrain data bases are common), the size of the terrain data base increases exponentially. This drives up the cost of development of the database for a given size area, as well as the cost of storing the database and using it in a modeling application.

An important determinant of interaction of opposing combat forces is their intervisibility. Intervisibility calculations made between two points on the ground using a given digital terrain database can provide approximations of how much of a given unit (such as an Abrams tank or an infantry squad) located at one point is exposed to visual contact by an opposing unit an another point. Such calculations are computationally expensive, generally involving algorithms that step along a vector (ray) between the points, looking for intervening terrain which would interrupt straight line-of-sight (LOS) between the points. The cost of computing intervisibility between many pairs of opposing units, as the units move across the terrain at time intervals, increases rapidly as the terrain resolution is increased.

The question to be considered for a combat modeler is, "How much terrain resolution is enough?" Some combat modelers insist that there can never be too much resolution. Other modelers believe that there is a point of diminishing returns. The answer probably lies in the modeling application. For driving

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detailed realistic visual depictions of moving units on actual terrain on a computer display, greater resolution may be necessary. For combat models requiring target detection and engagements in a computer simulation, there may be only marginal value in terrain resolution beyond some minimum threshold value.

We concentrate on the issue of the accuracy of intervisibility approximations as a function of terrain resolution. Many modeling studies for the Army have been done to attempt to characterize this resolution issue for combat modeling in general. This is an attempt to better characterize the issue in regards to intervisibility on the simulated battlefield. We also provide examples that relate back to military doctrine and tactics to motivate the user of this module.

Intervisibility On Two Dimensional Terrain

We begin by imagining a creature living on a two dimensional (*x* and *z*) terrain represented by the Cartesian graph of a function *f* over a domain *D* consisting of a bounded interval of real numbers. At each point *x* in *D,* define the "intervisibility", *I(x)*, to be the total length of the arc segments consisting of the points *(z, f(z))* for *z* in *D* that are visible from the point *(x, f(x))*. A point *(z, f(z))* is intervisible with *(x, f(x))* if there is no point *(v, f(v))* such that *v* is between *x* and *z* and *f(v)* is greater than the height of the line segment connecting the two given points at *v*. That is, for all *v* between *x* and *z.*

$$f(v) \le f(x) + (f(x)-f(z))(v-x) \, / \, (x-z).$$

In a computer modeling algorithm this condition is checked by comparing the three slopes between pairs of points selected from the two given points and the test point *(v, f(v))*.

A plot of points *(x, I(x))*, which we call the "intervisibility curve" provides global information about the intervisibility characteristics of the terrain *f*. For example, consider the terrain of a flat plain over a finite region with a v-shaped valley running through it. The inter-visibility curve of the cross-sectional normal to the axis of the valley is shown in Figure 1. Then *I(x)* (see Figure 1) will be a step function: for all x-values under the flat plain, *I(x)* will be the length of the arc associated with the plain. All points on the plain are visible from points on the plain, and no points in the valley can be seen from interior points on the plain. At the edge of the valley, *I(x)* jumps to include arc lengths of the valley since we can now see into the valley from the edge. At points within the valley *I (x)* becomes a constant value. This shows *I (x)* may have points of discontinuities (points where the derivative does not exist).
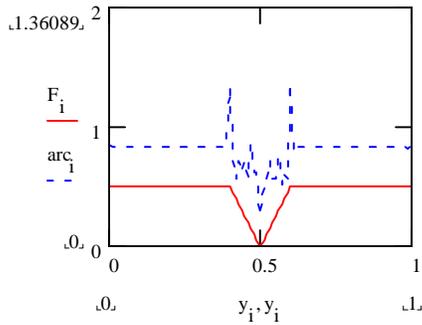
Intervisibility for Simple Terrain

Figure 1. Simple terrain model (Cross-section of a plain with a valley; show in bottom curve) and corresponding intervisibility curve (top curve).

As a second example, consider the function $f(x) = |cos(p\ x)|$ over the domain $D = [0,1]$. Then $I(x)$ starts at zero at the lower end point, increases to a maximum near $x = 1/3$, then drops rapidly to zero at $x = 1/2$ (see Figure 2). Symmetry of $f(x)$ about $x=1/2$ implies corresponding symmetry of I. Over $[0, 1/2]$, this is an example of what the Army Field Manual 21-26 calls "convex terrain." Note also that the maximum intervisibility occurs well down the slope from the crest of the hill at $x = 0$.

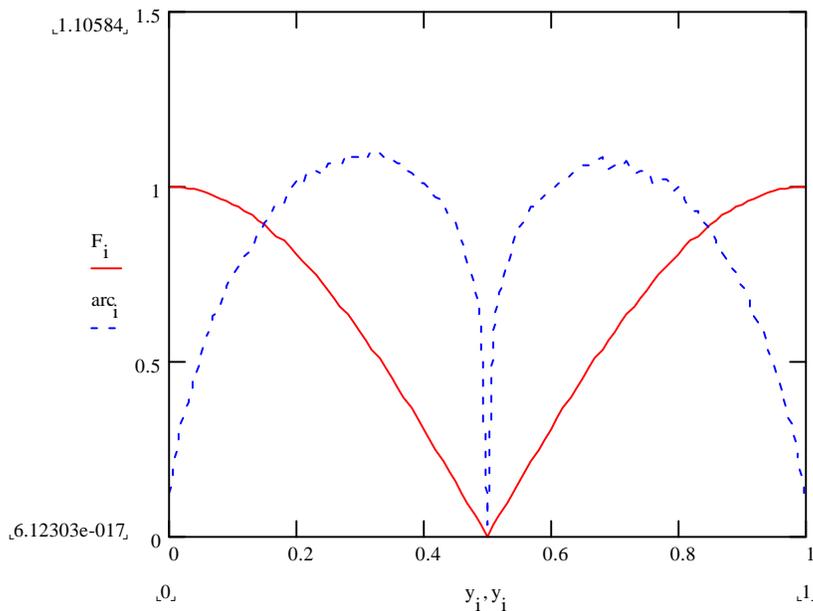## Absolute Cosine Terrain Intervisibility Curve

Figure 2. Intervisibility model (dotted curve) for one period of the absolute cosine terrain (solid curve). Note the maximum intervisibility occurs at x = 1/3.

Terrain Resolution and Computation of I *(x)*

In the case of this two dimensional terrain, digitized terrain can be characterized by the height of terrain at a finite set of points $X_n = \{x_0, x_1,...,x_n\}$, where n is a measure of the resolution of the map. We order the x's sorted in increasing order and assume they are equally spaced within D. The terrain map supported by the database, *{(x$_i$,f(x$_i$)): i=0,1,2,...,n}* is a polygon formed by connecting adjoining points of the form *(x$_i$,f(x$_i$)), (x$_{i+1}$,f(x$_{i+1}$))*; this polygon approximates the actual terrain *{(x,f(x)): x$\hat{I}$ D}*. The digital terrain map approximation of intervisibility at point *x*, *I (x)*, is then the sum of the lengths of the polygon segments visible from *(x,f(x))*.

Let's consider one of our previous functions, *f(x) = |cos(p x)|* over the domain *D* = [0,1], over two periods. The approximating intervisibility curves for *n*=20, 50, and 400 are shown in Figures 3 through 5. The most accurate approximating intervisibility curve model is shown in Figure 5. We note the *n*=10 is a crude approximation. Trying values of *n* > 100 did not significantly improve the resolution. Thus, we might conclude the resolution beyond n=100 is not cost effective to model this terrain's intervisibility.

Military Doctrine Involving Intervisibility

As long as soldiers have fought battles, combatants have sought to control terrain features that rise above the battleground. From these decisive points, a military force has a distinct advantage. Defending from the high ground required the attacker to expend additional effort to negotiate the upward slope while the defender concentrates solely on firing at the attacking force. From the high ground, an occupying force can see the battlefield from an enhanced perspective enabling the force to gain information about the enemy while denying it to the enemy. Early detection, for example, gives a leader an opportunity to seize control of the battle. The leader can gain surprise over an unsuspecting enemy or gain time to alter their course of action.

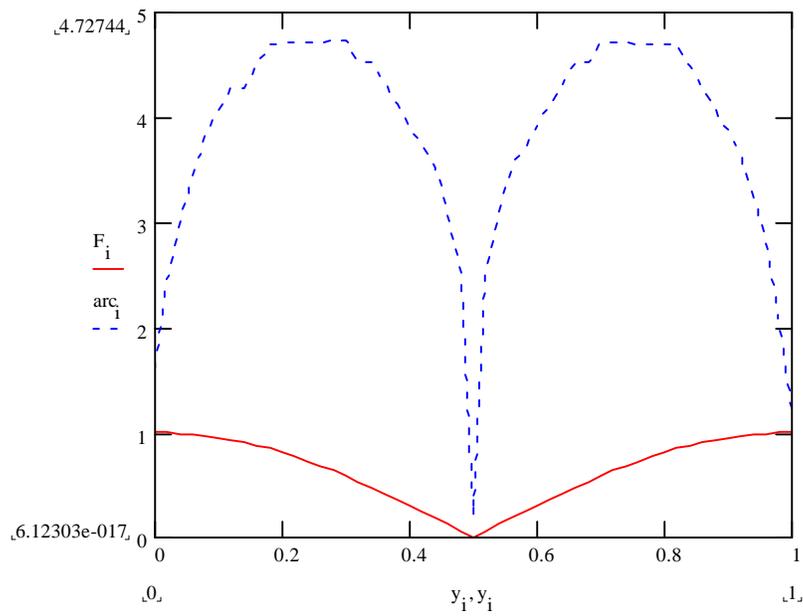**Absolute Cosine Terrain Intervisibility Curve**

Figure 3. Intervisibility with N = 20.
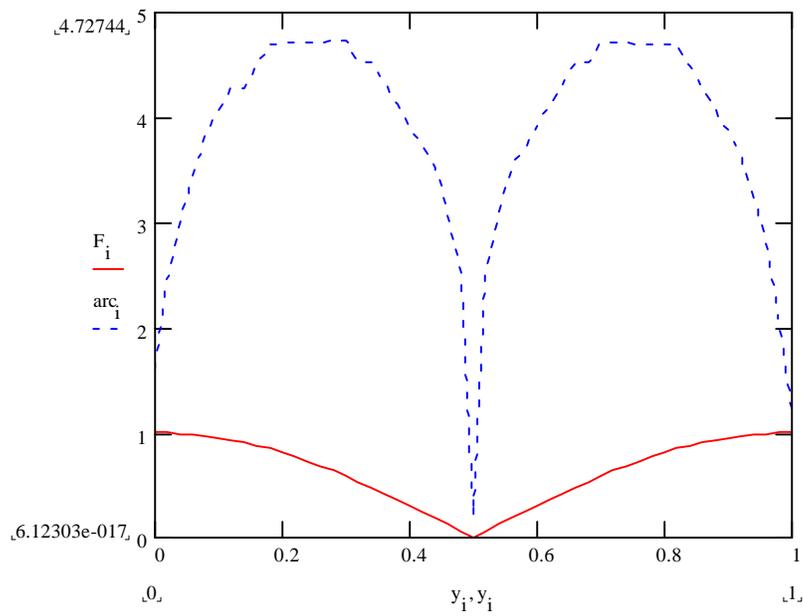

Absolute Cosine Terrain Intervisibility Curve

Figure 4. Intervisibility with N = 50.

Absolute Cosine Terrain Intervisibility Curve

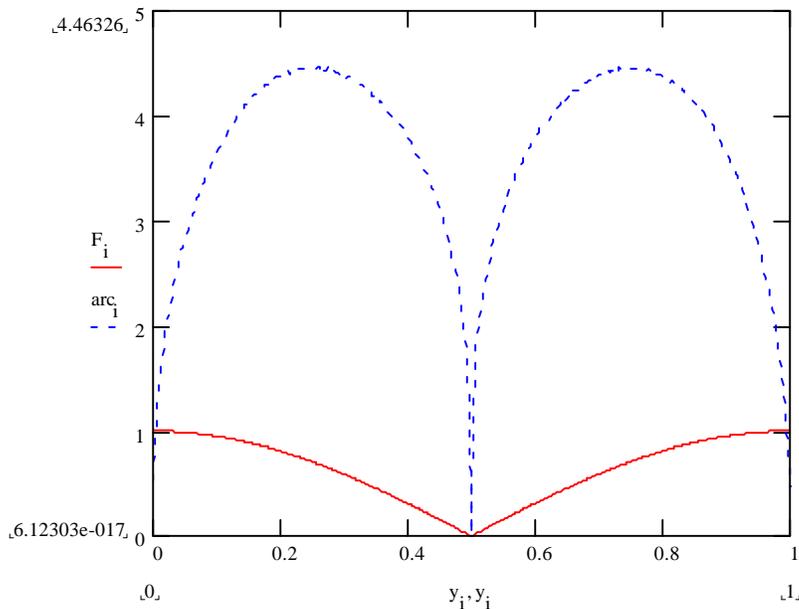
Figure 5. Intervisibility with N = 400.

The US Army has long understood this principle and has included it in its doctrinal literature. Current US Army FM 101-5-1, defines "key terrain" to be "...any locality or area that the seizure, retention, or control of which affords a marked advantage to either combatant."

Certain terrain is considered much more important than other terrain and is called "decisive" if it can have an extraordinary effect upon battle. It is essential to control decisive terrain because failure to do so will cause the force to fail in its prescribed mission.

Depending upon the particular mission, hilltops are frequently listed as both key and decisive terrain. During the Anzio campaign in WWII, the Alban Hills were the decisive and key terrain needed for the allies to succeed. Seizing those hills was part of the OPORD but it was not carried out. Once key terrain has been identified, the commander must further analyze the terrain features. The goal might be to maximize LOS or distance of unimpaired vision. By doing this, the commander allows the detection assets to operate at their fullest capabilities. It also allows weapons to be used at maximum possible ranges. There are two properties of hills that we are concerned with: the "topographical crest"--the highest point of a hill, ridge, or mountain; and the "military crest"--the area on the forward slope of a hill or ridge from which maximum observation covering the slope down to the base can be obtained.

The location of the topographical crest remains fixed at the highest point of the terrain; however, the military crest's location is dependent upon the type of hill slope. FM 21-26, has four classifications of slopes of interest to military operations. They are defined as: uniform gentle, uniform steep, concave, and convex. Gentile uniform slopes allow the defender to fire their weapons approximately parallel to the ground at a height of 1 meter. This type of fire is known as grazing fire and is considered to be the most effective type of direct fire. As the slope increases, the ability to use grazing fire decreases and becomes negligible when the slope reaches the uniform steep category. From a position on top of a concave slope, the defender can observe the entire slope and terrain at the bottom, but cannot use grazing fires. The topographical and military crests are at the same location in this case. The situation is different for a convex slope; the defender not only has limited chance for grazing fires but also has restricted LOS. The defender wanting to maximize LOS must move down to the military crest. These are depicted in Figures 6-7, respectively. Successful military operations are doctrinally dependent upon taking advantage of the terrain where the forces fight.

Calculations to find the topographical crest of the hill can be easily accomplished through models using calculus or numerical algorithms. The topographical crest is located where the first derivative of the function, $f'(x)=0$ and the second derivative of the function, $f''(x) <0$. For the function, $y = \cos(\pi x)$ over D=[0,1]. We find the first derivative $y'=\pi \sin(\pi x)$ and set y' equal to zero. This is satisfed at $x=0$. The second derivative $y'' = -p^2 \cos(p x)$ which is < 0 at $x=0$. Therefore the topographical crest of this function is located at $x=0$ with a functional value of 1.
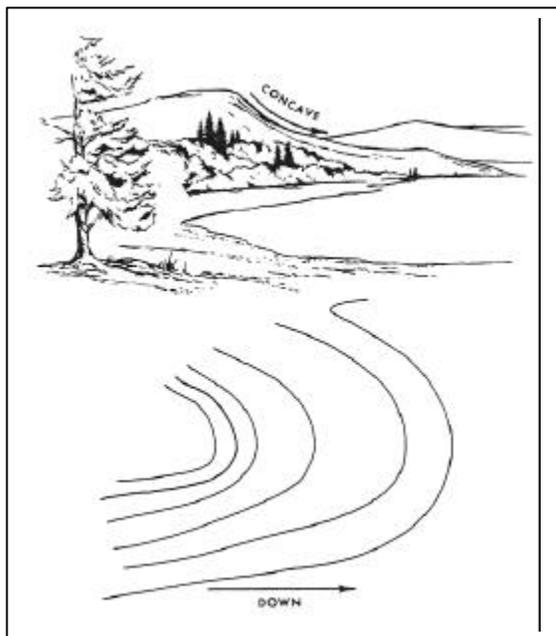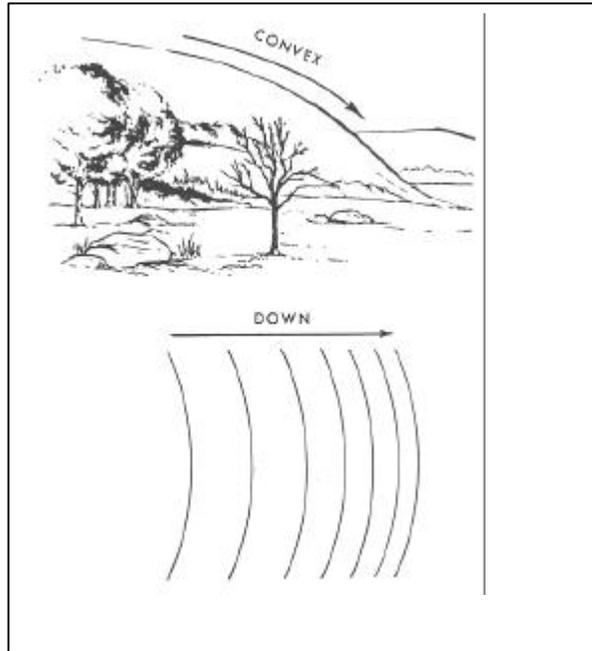
Figure 6. Concave Slope from FM 21-26 (Jan 1969)

Figure 7. Convex Slope from FM 21-26 (Jan 1969)

Now, consider our example function $f(x) = |cos(p\ x)|$ over the domain D = [0,1]. This function is not differentiable over the entire region and therefore, calculus cannot be used to find the top of the hill. In cases like this, a one-dimensional search method might be used to find the top of the hill within some tolerance. Many modeling methods exist (such as golden section and Fibonacci's method). We will illustrate the simple and direct method (not computationally as efficient as the others) of bisection. Bisection, like the other modeling methods, requires the function to be segmented into unimodal intervals. Unimodal means only one peak or one valley in a given interval. Thus, we will separate this function into intervals [0, 1/2] and [1/2, 1]. The bisection algorithm is shown below.

BISECTION ALGORITHM

Step 1. Select your unimodal interval [a,b] and a tolerance t (small like .02).
Step 2. Compute the mid-point (a+b)/2 and then offset be a small value, δ.
  Let δ be .01, so
  x_1= **midpoint - δ**
  x_2=**midpoint + δ**
Step 3. Find f(x_1) and f(x_2).
  (a) If f(x_1) > f(x_2), then let a=a and b=x2 and continue.
  (b) If f(x_2) > f(x_1), then let a=x1 and b=b and continue.
Step 4. If (b-a) < tolerance then quit, otherwise find new midpoint and

repeat steps 2 and 3.

Results

a	B	Midpoint	x1	x2	f(x1)	f(x2)
0	0.5	0.25	0.24	0.26	0.72923	0.684849
0	0.26	0.13	0.12	0.14	0.929847	0.904922
0	0.14	0.07	0.06	0.08	0.982305	0.968615
0	0.08	0.04	0.03	0.05	0.995566	0.987701

We conclude that in the interval [0, 1/2] the maximum lies in the final smaller interval of [0, 0.08]. By changing the tolerance levels, we can make the final interval arbitrarily small.

The calculations of "intervisibility curves" shed some light upon the location of the *military crest.* In our previous example, we noted that the maximum intervisibility occurred about 2/3 of the way from the crest to the bottom of the valley, well away from the topographical crest.

Exercises

Use your own program or the MATHCAD template from the appendix to solve these exercises.

1. Find the topological and military crest for a terrain characterized by $f(x) = |Sin(3px)|$ over D=[0,1].

2. Find the topological and military crest for a terrain characterized by $f(x) = |Cos(3px)|$ over D=[0,1].

3. Find the topological and military crest for a terrain characterized by $f(x) = |Sin(3px)|(1-x^2)$ over D=[0,1].

4. Find the topological and military crest for a terrain characterized by $f(x) = (1 + \sin 6\pi x))(1-2x/3)$ over D=[0,1].

5. Describe how you as a platoon leader can use these tools to help in your military decision making.

6. What over applications or use might there be for intervisibility? Briefly describe the application and the use of intervisibility.

7. Develop an algorithm to compute intervisibility in higher dimensions.