EDITOR’S NOTES

Thank you to all of the authors for prompt submission. It is a hectic time of year. That so many took the time to write indicates the importance of the issue at hand.

“I like the article by LDCR Skufca…it almost convinces me to drop the calculator from my Calc 1 classes, and I usually come down closer to the pro-technology side of the debate…”

“Seeing the problems worked out on the board, and performing them myself…I was still unable to completely grasp the purpose of many of the lessons. The information started to make sense and become much easier when I learned to utilize the…calculator…”

This is more than just an interesting debate about pedagogical styles and personal preferences. Everyone will agree that the intention is for the students to gain understanding, become “real world” problem solvers and to obtain the desire to learn more. The hope is that technology will aid in accomplishing these goals. *Is this the case?* It is time to evaluate ourselves. Part of this process is to carefully read this semester’s articles to assist in finding answers to “What is appropriate use?” “What is inappropriate use?” “What are the levels at which students can utilize technological devices?” “How do we decide whether to use technology and, if so, at what level?” “Does the overuse of calculators in the beginning math courses hurt our students’ ability to master skills and **critically** **<u>think</u>**?”

While it is essential that our graduates be knowledgeable about modern technology and comfortable in the utilization thereof, it is critical that we, as educators, ensure that our students are using the technology as a tool rather than a crutch.

Best wishes from West Point,
Mary Jane Graham

CONTENTS

SUBSCRIPTIONS TO *MATHEMATICA MILITARIS*:

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Make Way for Technology!

LTC. Philip Beaver, USMA

The problem of dividing four by seven is not unique to the United States Military Academy. As curricula become more diverse and specialized, all academic departments must compete for a monotonically non-increasing slice of the pie. This is especially true for "service" departments, particularly the Department of Mathematical Sciences, which typically has fewer than thirty majors, yet teaches at least four semesters to all 1100 students in each entering class. The seven-into-four solution includes filling the four core semesters with discrete dynamical systems, single-variable calculus, multivariable calculus, differential equations, probability, statistics, and linear algebra. Fortunately, all of these topics can be streamlined by teaching them with technology, so each now fits snugly into its respective box. Or perhaps not.

Every cadet comes to math class armed with a graphing hand-held calculator (HHC), and they all have computer algebra systems (CASs) in their rooms. This allows for many skills, which were previously done manually, to be done with technology. The time savings created by not having to work manually, or to flip through cumbersome tables, are not lost to the time spent pushing buttons or loading systems; however, we do lose some time to the non-trivial ramp-up training on this technology. This leaves some tough choices when slicing the "traditional" approach to teaching these courses to both make way for technology and to fit them into their boxes. When making these choices, it is too often the case that rigor is the first thing to be cut.

Most of the mathematics we teach can be taught with technology, which only makes sense, since most mathematics today is done with technology. However, there are many skills that should be learned manually in order for the students to gain a fundamental understanding of them, above and beyond learning which button to push on the HHC. This requires a detailed look at our curricula to determine where each topic fits into the "technology hierarchy."

We can divide our curricula into four subsets where each indicates a different technological emphasis. While these subsets are not entirely disjoint, each course topic or learning objective can (and should) be placed in one before it is taught. The subsets are as follows.

1. Topics or skills that students must learn manually (with no assistance from technology at all).
2. Topics or skills that students should learn manually so that they can perform them with technology.
3. Topics or skills that students should learn manually only to gain an appreciation of the technological solution.
4. Topics or skills that should be taught exclusively with technology.

Each one of these subsets has its place in the mathematics curriculum, and almost every educator would agree that none are empty. However, there is a lot of disagreement on how they are filled. Every department has a few dinosaurs who feel that if it was good enough for them, it's good enough now, and while very few mathematicians still succumb to the urge to interpolate on log tables, there are a number who feel that calculators should be restricted to five keys (+, –, *, /, =) until graduation. We discuss each of these subsets in turn, and offer a few examples of where various lesson objectives might fit in.

The first subset, non-technological topics, is where we find "the basics." Occasionally it's O.K. for an instructor to lecture to his students, and while this does not allow the students to be guided down the path to discovery, we must sometimes make this sacrifice. (A beneficial, yet overused, offshoot of technology is to have students experiment with the available tools to "discover" concepts on their own.) This subset includes definitions (how does one "discover" a definition?), theorems and their proofs, and perhaps some modeling. It is possible that entire analysis or topology courses may still be taught without technology, but as we are focusing on the four core semesters, we will consider more basic examples. It is still useful to derive the definition of the derivative, to integrate Riemann sums into a lecture, and to provide an array of matrix and vector results without ever pushing a button. A few fundamentals (such as a theorem of Newton's from calculus) and central ideas (perhaps a limit theorem from statistics) should probably be discussed exclusively with chalk before the students are allowed to assault a computation with a computer algebra system. However, once we begin to apply these definitions and basic results, we begin to blur into the "technology" subsets.

The second subset, those skills that must be learned manually in order to perform them electronically, include some of the most fundamental computational ideas from all of our disciplines. The chain rule for differentiation, u-substitution for integration, matrix multiplication, and computing areas under the normal distribution curve are all easily tackled by any good HHC or CAS; however, the student who does not understand the principles underlying these concepts has no need to ever turn on his calculator. These are concepts that must not be allowed to perish, as they form the building blocks for all future work, and their theoretical importance far outweighs their roles as computational tools. There are those who will insist on expanding this subset to include various differentiation techniques (products, powers, quotients), several integration techniques (trig substitution and integration by parts), but for many these have already slipped into the "nice-to-know" subset and off of the lesson objective list.

The third subset contains those tasks we expect students to perform exclusively with technology, but which should be performed at least once manually to gain an appreciation for the process their CAS or HHC uses. This includes finding the inverse of a matrix (perhaps up to a 3×3), calculating a determinant (perhaps of a 4×4), computing eigenvalues and eigenvectors, computing the slope and intercept in a simple linear regression model, differentiating a product, quotient, or trigonometric function, and finding a definite integral. It may also be useful for students to compute an integral involving a trigonometric substitution at least once, to avoid bewilderment the first time they integrate a function with $1+x^2$ in the denominator, only to have the calculator return something involving the inverse tangent function. This category also includes most of the arithmetic our students learn in elementary school, finding roots of equations, calculating Fourier coefficients, and computing probabilities from most standard distributions.

The fourth subset includes all of the subjects or topics that may be taught exclusively with technology. While some teaching-with-technology proponents may suggest this includes our entire core curriculum, we should perhaps approach this subset with a little judgment. A strong argument can be made that once the basics of differentiation and integration have been learned, all further computations should be done with technology. The product and quotient rules, integration by parts and trigonometric substitution, finding logarithms, powers and square roots, and most skills that used to be done via tables or charts, can now be calculated with the touch of a button. Furthermore, many numerical methods, simulations, and numerical demonstrations (such as of the Central Limit Theorem) should be done entirely on the classroom computer. Here we find a lot of "techniques" that developed out of necessity over the years that may now be obsolete because of technology. As technology improves, this subset continues to grow, and many see it as all encompassing in the not-too-distant future. However, due to the shortsighted view many take of this subset, this is a particularly dangerous trend.

The loss of rigor many curricula are experiencing comes from too many subjects being placed in this fourth subset, when they really belong in the first. We frequently fail to look outside the confines of our own courses and the subjects they immediately support in this regard, and this is where we are letting our students down the most. This danger can be avoided if we consider the "bigger picture" when designing our curricula. No one would argue that there is ever a need to again teach high school students how to interpolate on log tables. However, has the basic idea of interpolation been lost forever from our preparatory curricula? Do students now encounter interpolation for the first time when they build a model with linear regression, some time after studying multi-variable calculus? Is it when they study convex functions in analysis, or convex spaces in topology? Hopefully, our high school physics courses have saved a place to formalize this most basic idea.

We would like our students to understand why the complex eigenvalues of real matrices always occur in conjugate pairs, but if their understanding of the quadratic formula and the determinant has been reduced to locating a button on the HHC, this concept will be out of their reach. A favorite topic for the chopping block is integration by parts, since the HHCs seem to understand it completely. However, what does this do for the student who some day may want to solve a boundary-value problem with a Green's function, to compute the asymptotic expansion of an integral, or to use perturbation theory for approximating eigenvalues? It is easy to brush this off as something they will pick up later if they go that far in mathematics or engineering, but if not in the core courses, then where? Of course, including integration by parts brings the product rule for differentiation back into the program as well, and suddenly we don't have enough room in the curriculum for the students to discover Euler's method. Four no longer seems quite so divisible by seven when doing it with technology.

These remarks need to be put into perspective by considering our target audience. We are preparing most of our students for undergraduate engineering degrees, and some will take little mathematics beyond the core requirements. It is true that many will pursue Master's degrees or higher in some technical discipline, but we are more inclined to focus on the immediate mission. We primarily would like our students to depart the mathematics curriculum as competent, confident problem solvers, which is the best we can do in terms of giving them the education they deserve. However, we must be careful to not place too much stock in the power of sophomore calculus: how many of our "applications" are meaningful, real-world problems, and how many are contrived, scenarios that represent a futile attempt to show immediate relevance where perhaps none exists? There is a fine line between solving problems and "doing word problems." Unfortunately, in the shuffle of our lean, lively, teaching-teachers-to-teach-with-technology curriculum reform, many of our most deserving students have been handed a significant disservice. This loss of technical rigor "because the technology takes care of it for us" is not a positive trend.

Of course, there are more persuasive arguments for preserving rigor in the face of technology in our core curriculum, but simply citing the curvature of the Hubble mirror and the altitude of the Mars orbiter would have reduced this paper to a single paragraph.

Teaching and Learning with Technology

Dr. Mary Ann Connors, USMA
Dr. Edward A. Connors, USMA
LTC. Kathleen Snook, USMA

The fundamental criterion in evaluating the success of any course is how well the students grasp key concepts. For example, in a differential calculus course, we want students to understand that both the instantaneous rate of change at a given point in time and the slope of a line tangent to the graph of a function at a specific point are represented by a value of the derivative of some function. Additionally, we would like students to recognize that the derivative of a function is itself a function and, furthermore, that these functions provide much information about each other.

It is our belief that the appropriate use of technology can enhance teaching and learning. In particular, in this paper we will discuss specific examples where computers or hand held calculators with computer algebra systems (CAS) can be an aid in developing and reinforcing the comprehension of mathematical concepts.

What is inappropriate use? We believe that using the technology as a black box, simply pushing buttons to get results without understanding, is certainly ineffective in learning concepts. Also, it makes no sense to take the time to use the technology to do some elementary computations that one can do accurately and quickly by hand. A third misuse is to blindly accept all results without knowing or questioning the limitations of the technology. Finally, it is inappropriate to use technology merely for the sake of using technology.

What is appropriate use? As stated above, technology use is appropriate when it enhances teaching and learning; specifically, when it enables students to comprehend key concepts. We believe that the use of technology as an effective pedagogical tool should be helpful in presenting and/or learning a concept, as well as in reducing the time and effort involved in tedious and lengthy computations.

When we can't illustrate a complex graph by hand it is often helpful to use a calculator or computer with an overhead display unit. Instructors and students can interact with the technology. For example, sequence plots, cobweb plots and phase portraits of discrete dynamical systems - particularly for non-linear models - are easily done with some calculators or computer software. The presentation of Riemann sums is enhanced by the visual display of graphing devices. Slope fields of differential equations can be plotted readily with some hand-held technology. In a multivariable calculus course 3-D, implicit, or contour plots can be produced with the stroke of a few keys.

When we have lengthy or complicated computations to perform, it is more efficient to use technology. Calculators and computers offer algebra systems and symbolic solvers that dramatically reduce the amount of by-hand number calculations and symbol manipulations. Derivative and integral problems requiring multiple steps by hand become straightforward when using technology. We easily compute iterates of a recurrence relation using technology. In linear algebra, we quickly find determinants of 3x3 or higher matrices, as well as the eigenvalues and eigenvectors of those matrices. We then use the time saved by these efficiencies to analyze the results and meanings of computations. Technology has allowed a refocusing on conceptual understanding versus by-hand procedural proficiency. Although procedural proficiency must be maintained, students who were previously bogged down in numbers and symbols can now lift their heads to see the broader concepts.

It is possible to simulate a laboratory environment in many mathematics and science classrooms by collecting and analyzing data. Solving various types of practical applications is made possible with data collection devices and calculators. Once data is collected students can analyze the data to look for patterns and behaviors. Similar activities can be done with data obtained from other sources or from data generated by complex functions that are difficult to analyze analytically.

Technology allows students to move among the three representations offered by the technology; graphical, numerical and analytical. When we want students to see patterns we can lead them to investigate by observing the effects of changing various parameters. They can see the effects of those changes in all three representations. For example, students can change the algebraic definition of a function, observe translations and phase shifts in their graph, notice changes in their data tables, and then, generalize and make conjectures.

There is evidence that appropriate use of technology does help students to learn some topics better. Some studies which provide examples of the use and effectiveness of technological pedagogical tools are included in Connors, 1995; Dunham, 1998; and Huley, Koehn, &Ganter, 1999.

We cannot know for sure what will help a particular student learn better, but we can increase their chances by providing an environment where they can explore, discover, confer, test and validate with the best tools available. We know something helped a student to learn and understand when they tell us in their own words.

Here are some quotes from students that speak to how the use of technology enhanced their learning about discrete dynamical systems.

Better Conceptual Learning

"We spent the last part of the block learning about how to create and use a cob web graph. I enjoyed this area not only because it looked cool but also because it successfully brought together several concepts together and put them into a manner in which I learn best, visually."

"My calculator was my saving grace when it came time for me to test. … Finding everything long hand, items like eigenvalues and eigenvectors, would have taken forever and would have led to more confusion. The TI-89 made my life exponentially easier, and that is what led to my eventual comprehension of the material taught in block two."

"The use of a calculator makes the solving process go faster when the proper commands are used. The calculator is a powerful tool for helping me understand a concept …"

"Seeing the problems worked out on the board, and performing them myself in the nightly drill problems, I was still unable to completely grasp the purpose of many of the lessons. The information started to make sense and became much easier when I learned to utilize the TI-89 calculator in solving the various problems. By visualizing the actual results obtained, the material began to make sense and seem worthwhile."

Determine patterns to better understand

"This use of this technology enables me to enter a matrix, perform the various operations, and graph my results. By having a graphical representation of the equations, I can explore the different characteristics of the equations and determine patterns to better understand the system."

Greater ease in finishing assignments and checking answers

"I discovered that both the TI-89 and Mathcad-8 computer program are very useful tools in solving math problems. The calculator helped extensively on eigenvalues and eigenvectors during assignments and the WPR. The computer program offered superb help with the math project we were assigned to complete. Through the use of these two resources, I was able to finish my assignments with much greater ease.

As part of our study I also learned how to use technology to aid in finding solutions. While I'm not an expert at Mathcad, I was able to use the graphing calculator to help me solve most of the matrices."

Made math life more interesting by enriching the mathematical experience

"I also learned many mathematical applications for the TI-89 calculator, which made math life much more interesting! An example of my newly acquired calculator skills is the use of a cobweb plot. Calculator is the most efficient way to iterate systems of DDS's. You can use either iterations or cobweb plots to determine the long-term behavior of a system of DDS."

Assistance in solving a complex problems and confirming its validity

"I also learned a great deal from the project that we did during this section. This project was able to tie together everything that we had learned in order to solve a very complex problem. We had to incorporate such techniques as finding the solution in both matrix form and through eigenvector decomposition. We were also able to see how eigenvector decomposition can predict the long-term behavior of systems. This project also demonstrated to the class how invaluable such tools as mathcad and our calculators are. Without these tools, we would never have been able to solve the problems. Mathcad is able to do everything the calculator does and is able to print graphs and tables of the results. Another feature that I like is using both mathcad and the calculator to solve the problem, and then checking the results of each to verify that my solution is correct. Mathcad proved to be an invaluable tool when it comes to making nice presentations of results."

Lifeline

"The last few sessions that we had this section was devoted to learning how to manipulate cobweb graphs in order to determine both the long-term behavior and the stability of the system. We were able to see how different systems work and how they look in cobweb graphs. We looked at the graphs of nonlinear DDS's with both stable and unstable equilibriums as well as linear DDS's with stable and unstable equilibriums.

The TI-89 calculator was very useful in this block. We learned how to solve a DDS in matrix form, find eigenvalues and eigenvectors, and form cobweb plots. The cobweb plots helped us to determine the long-term behavior of a DDS. … The calculator was basically our lifeline for this block. We worked a lot with learning how to do things on the calculator because it would take too long to write out by hand."

Powerful technology - Key to success - Practical Applications

"The calculator was key to success in this block. Although the book showed us how to use each method by hand, I would have wasted countless hours and sheets of paper trying to do it without technology. With a few strokes of the keys on the calculator your answer appears on your screen. I was definitely grateful to have this powerful technology for my use.

It was also convenient to be able to graph the functions on the calculator. All I had to do was type it in to the y equals menu then it would graph it for me. I could also get a table also with the same technique. Both the graphs and the tables serve to show what the system is doing and where the equilibrium value is. The table is more specific because you can see the actual numbers as they approach equilibrium.

Over all this has been an interesting block, because it taught practical applications when using matrices. It always helps to understand how you can use things in real life. … I feel like I could use this for many things in the real world."

As technology becomes more and more accessible and prevalent, the ongoing debate on the effectiveness of the use of technology will continue. We hope that these remarks are helpful in recognizing some of the benefits of appropriate use of technology.

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Laptop Computers in the Classroom

MAJ James A. Glackin, USMA

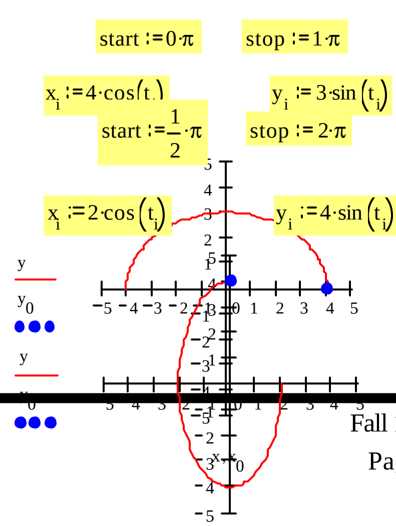
The United States Military Academy (USMA) is currently conducting an experiment with laptop computers. Thirty-two cadets were issued laptop computers for use in MA205, Multivariable Calculus and SS201, Economics. They were also asked to use their laptop in the barracks instead of their desktop computer. While the Academy focused on deciding if the laptop is a viable alternative to desktops, the Department of Mathematical Sciences focused on how to use this additional technology in the classroom. Particularly, how could we use Mathcad to increase cadet discovery and understanding? We also wanted to find ways this experiment could help those without laptops.

Every classroom at West Point is equipped with a computer and projection device. Often during class instructors use Mathcad to help teach cadets how to use the program and to aid in visualizing a problem. Although better than nothing, this technique left several cadets very unfamiliar and unconfident in their ability to use technology to help solve problems. The reason for this was the time delay between seeing things in class and trying to execute the commands back in the barracks. When cadets got back to their rooms they had often forgotten the required syntax.

With laptops in the classroom there is no time delay. Cadets watch me perform operations on Mathcad and can follow along, ensuring they get the same results. If they don't, they can immediately ask me to take a look at their worksheet. The difference is evident in student attitudes, "Before I saw the use of Mathcad as a chore and was even afraid to use it… Now I am not only more confident with Mathcad, but I enjoy using Mathcad and see the advantages."

Although my initial focus was to make cadets confident in their ability to use Mathcad, I quickly realized this was an intermediate step. What I really wanted was for cadets to better understand Multivariable Calculus by using Mathcad to perform tedious time consuming calculations or to aid in visualizing a problem. To do this I would create a worksheet and email it to cadets the day before class. As part of their assignment I asked them to "play with" the worksheet before class. During class they could ask questions and I would bring out the concepts I wanted to emphasize. This technique can also be used with students who solely have desktops.

Parametric Equations: When we study parametric equations we spend a few minutes learning the concepts, then spend several hours doing "stubby pencil" work. By using worksheets Mathcad did the "stubby pencil" work in seconds. Cadets then use the extra time to explore what affects changes in the parametric equations have on the plot. Here are a few simple examples changing

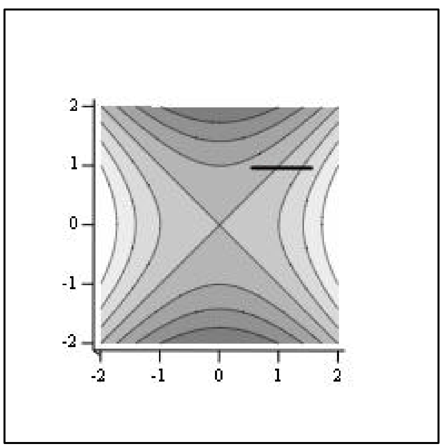


the coefficients, start, and stop points. Cadets can do dozens of changes in minutes.

After making a few changes on their own cadets are able to predict what the plot will look like before it is generated.

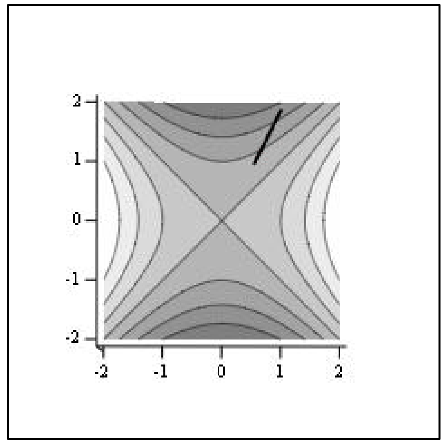
Directional Derivatives: Another area where experimentation is very useful is the directional derivative. Cadets quickly learn how to compute the directional derivative, $D_{\vec{u}} f = \vec{\nabla} f \cdot \hat{u}$. Very few understand what the numerical answer represents, the rate of change in the direction of motion. Armed with a Mathcad worksheet they can change a starting location or direction and immediately see the impact. In the following contour plots each contour line represents an elevation change of one. Darker shaded regions are lower.

$x0 = 0.5$ $\quad y0 = 1$ $\quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad Duf = 1$ $\quad maxDuf = 2.236068$



(x, y, z), (X, Y, Z)

$x0 = 0.5$ $\quad y0 = 1$ $\quad u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad Duf = -1.341641$ $\quad maxDuf = 2.236068$



(x, y, z), (X, Y, Z)

Again, after experimenting with the worksheet cadets are able to predict what the answer should be before it is computed. They can also experiment and discover the maximum directional derivative is achieved when we travel in the direction of the gradient.

These are just a few examples of how technology can help students better understand Multivariable Calculus. I do not know if USMA should issue every cadet a laptop. I do know there are advantages to having a laptop in the classroom. But we can also realize some of these advantages with the technology we currently have. What we all must understand is technology does not replace learning; it should enhance it. I believe that is what we have accomplished.

Insights Regarding the Incorporation of Technology Into the USAF Academy's Mathematics Curriculum

Lt Col Steven M. Hadfield, USAFA

Technology in the mathematics curriculum at the U.S. Air Force Academy has always been an area of emphasis but is especially so now with the vast array of technology-based resources available. In this article, we summarize what we use technology for, what are the primary technologies we use, the levels at which technology can be utilized, and what's required for the effective use of technology. We also provide some key pro's and con's to the use of technology and conclude with some questions to ask regarding whether or not to incorporate technology and, if so, to what degree.

<u>What can technology be used for in mathematics education?</u>

We typically organize the use of technology in mathematics education as computational, illustrative, or explorative, however these are not distinct categories nor are they comprehensive. Computational usage most frequently involves using the calculator or computer to perform some computational task. In the past, this has been mostly numerical (arithmetic) calculations but with current computer algebra systems (CAS) on both computers and calculators, this now includes all types of symbolic manipulation operations. Computational capabilities can be used to verify hand calculations as these skills are being learned or to complete more extensive operations that would be tedious and error-prone to conduct by hand. Technology-based activities that "drill" the students on particular techniques would also fall within this category.

Illustrative use of technology for mathematics most frequently involves the use of graphics and animations to illustrate key mathematical concepts and to help develop mathematical intuition. Examples of the usage of technology for illustrative purposes could be an animation of secant lines between two points on a function as one point approaches the other to illustrate the limit definition of a function's derivative. Another might be the animation of a particle traversing the unit circle with real-time plots of the associated values for the sine and cosine functions corresponding to that point. Illustrative uses tend to be most appropriate in the classroom setting but can also be effectively used elsewhere.

Explorative use of technology involves utilizing the technology in close concert with creative and critical thinking skills to quickly answer "what if" type questions regarding a particular topic, application, or integration of topics. The emphasis here is on the student's freethinking and problem solving skills using the computer to support answering the questions that students generate in their exploration. Explorative activities with technology can be the most rewarding but are also the most difficult to design. They can range from carefully contrived scenarios and tasks to lead the students to some discovery or to very open-ended activities where students are left to find their own paths.

Certainly it is obvious that a particular exercise might include more than one or all of these types of technology usages, but these categories are not comprehensive. For example, we currently have a fairly sophisticated database of test items for all of our core and service mathematics courses. The database holds the test items, key reference data for retrieval, and past performance data. With such technology, we can track student performance trends, which is a very valuable assessment tool that we especially emphasize with final exams. Here technology plays a supporting role in the background but nonetheless significantly benefits the effectiveness of the mathematics education.

<u>What are the primary technologies in use?</u>

In our calculus-based core and engineering mathematics courses we emphasize the use of the Mathematica CAS software for computational, illustrative, and explorative purposes. However, we also make use of current calculator technology including the CAS technology in the TI-92, TI-89, and HP-49 calculators. Furthermore, Capt Frank Wilson has spearheaded the use of animated PowerPoint presentations in the classroom as well as interactive games based on PowerPoint and embedded hyperlinks using a Jeopardy motif (see his article in this issue). He is also experimenting with fictional mystery stories with embedded mathematical problem solving to both motivate and captivate students especially at the more developmental levels. Capt Bob Clasen and Major Kevin Yeomans have been experimenting with the use of web pages for scenario-based problem sets and as repositories of course materials. Furthermore, we are developing a web-based repository of Mathematica notebooks that provides demonstrational, tutorial, illustrative, and explorative examples of how Mathematica can be used across the spectrum of undergraduate mathematics (see www.usafa.af.mil/dfms/mma.htm). We also make prolific use of shared network drives on our local area network as well as email for sharing lesson ideas and resources both between the faculty and with our students. Other courses in statistics make extensive use of Excel and Excel-based packages. Our numerical analysis course has emphasized the C programming language on both Windows and Unix platforms. While we have not made use of on-line chatrooms, they can also be effective technologies to support mathematics education, especially if they support mathematical symbology and graphical contents.

What are the levels at which students can utilize technological devices?

Within the past year, we have developed a taxonomy of the levels at which technology can be emphasized and used by the students. Our current model employs four levels, each with their own objectives, methods, and assessment techniques. These levels are Motivational Demonstration, Guided Employment, Selective Competence, and Comprehensive Mastery.

Motivational Demonstration is characterized by exposing the students to the capabilities of new technologies; sparking their intellectual curiosity; and instructor-based computational, illustrative, and explorative use of the technology. This is most typically accomplished by instructor usage in the classroom setting. Carefully constructed classroom activities with the technology can significantly motivate the students towards further usage and exploration of the particular technology and is an important prerequisite to advancing to the higher levels of employment. Assessment of this level of usage is difficult and typically limited to surveys and anecdotal measures. Interestingly, this level is sometimes all that is needed for the students to embrace the technology. We have chosen to pursue the new CAS-capable calculator technologies only at this level, but our students' familiarity with calculators combined with their desire for an advantage and the portability aspect of calculators have caused many students to further pursue and master these devices.

Guided Employment includes objectives of getting the students to be able to accomplish certain well-defined tasks with the technology given procedural guidance directing them as to how to do it. To accomplish this level, we need to provide in-class demonstrations as well as out-of-class tutorial resources. We also typically include some required usage of the technology to accomplish tasks, either in and/or out of the classroom setting but with allowing the required references. Evaluation of the students' use of the technology provides a viable assessment mechanism. Our core calculus courses emphasize the use of Mathematica at the Motivational Demonstration and Guided Employment levels.

Selective Competence requires that the students master the technology to be able to solve specific classes of problems in the technology utilizing no additional reference material other than what is available inherent with the technology. In additional to the methods used for the earlier levels, we must employ in-class, controlled usage of the technology with restricted external assistance and the scope of tasks that must be mastered needs to be clearly defined. Assessment mechanisms include in-class, hands-on evaluations together with possibly exam questions on the technology. We are currently employing Mathematica at the Selective Competence level for our series of three engineering math courses in multivariable calculus and differential equations.

Comprehensive Mastery requires that students be able to employ the technology to solve problems across the spectrum of the technology's capabilities. To accomplish this level, there must be extensive instruction on the technology together with exercises where the students must employ the technology in ways not previously demonstrated by the instructor. Assessment mechanisms need to include controlled evaluations which exclude external assistance and require the student to employ the technology is manners not previously demonstrated. We do not currently pursue this level of employment in any of our courses, but we do strive for this level with the training provided for our instructors.

What is required for the effective use of technology?

In order to utilize technology effectively, we have found that there are a number of issues that must be addressed. First you need to motivate the use of the technology for both your students and your faculty members. They need to see that there will be some benefit for them taking the time to learn how to use the technology. Methods for doing this will vary by technology and some technologies will be much easier to motivate than others.

Second, you need to provide adequate training and reference resources for both the students and faculty. Again the techniques will vary with the technology, but there must be dedicated time for the training and some evaluation mechanisms to insure that the training is adequately accomplished. Furthermore, it is imperative that the training involves some "hands-on" activities with the technology, which may require some computer lab support. Repetitive usage of the technology is also a great benefit here and is well illustrated by Dr Beth Schaubroeck's technique of "a command a day" usage of Mathematica in our pre-calculus course.

Motivation and training are important, but they do not insure that all the students really "get into" the technology. To further advance the students' capabilities with the technology, we need to employ some type of evaluation mechanism. That is, we need to test their level of competence in a controlled environment where each individual is assessed. Such evaluations can sometimes be difficult to implement, as they are dependent on the reliability of the technology. For example, we have incorporated in-lab quizzes on Mathematica into our engineering math courses that have the students open up electronic notebooks (files) that specify tasks to be accomplished. The students then accomplish those tasks in their copy of the notebook and either print it or save it to diskette for turn in. With a migration to a new network operating system, our current network print server configuration can not handle the print load and a bug in Mathematica causes 5-10% of notebook save operations to result in corrupted notebooks (lost data that precludes Mathematica from being able to properly display the notebook contents). As you can imagine, this has been a serious challenge. Our current approach is to carefully craft problems given on paper that require Mathematica to solve them in the time allowed and then the students simply write in the solution/answer so that their on-line work need not be saved nor printed. To assess plotting skills in this approach, we make use of matching type problems that require the student to accomplish various graphs with Mathematica to answer the question posed.

Nonetheless, we've found that effective teaching of the technology requires al three of these first three requirements to be addressed and use the phrase, "Show, Practice, Test" to emphasize their importance.

A fourth requirement is that of appropriate technical support including appropriately configured and maintained computer labs. Students must have access to the technology either in their rooms or in common use labs. Instructors need access in both their offices and in the classroom to include projection devices in the classroom settings. There should also be some lab space for student use of the technology in an in-class setting. Such labs would ideally have one computer per student arranged such that the instructor can view all the screens from a single location. There should be an instructor computer with projection capability as well as abundant chalk/white board space visible to all students.

I personally believe that it is imperative that the instructor emphasizes both the strengths and weaknesses of the technology. We need to take care to not develop a "blind acceptance" but rather a "healthy skepticism" toward technology where students utilize several methods to check and cross-check solutions.

Finally, there should be an appropriate availability of reference material supporting the technology. This can come in the form of on-line and text references available with the technology, in-house developed demonstrational and tutorial material, as well as third party products. Ideally, these reference materials are available on a long-term basis with mechanisms that make them easy to locate. Currently we use our web-based repository of Mathematica notebooks and shared network disk drives for such repositories. We also have created a hyperlinked set of Word documents (appearing much like web pages) that provide an easy to maintain hierarchy of menus for our faculty to locate a wide range of on-line references, both pedagogical and administrative in nature.

<u>What are the pro's and con's of using technology?</u>

Incorporation of technology into undergraduate mathematics education poses many benefits to be reaped, but there are also significant costs and risks. Below is a summary of some of the key pro's and con's to using technology together with some key decision criteria to be considered when deciding whether and how to incorporate technology.

PRO's:

- Students can be empowered to work more efficiently and accurately.
- Instructors can better illustrate topics for conceptual understanding by the students.
- Instructors can foster explorative, creative, and critical tendencies in their students.

- Instructors can move their students to higher levels of critical thinking sooner.
- Via repositories of existing material, instructors and students can exploit reusability.
- Entertainment aspects of technology can keep students more engaged in the learning activity, which is especially true for remedial students.

CON's:

- Instructors and students must devote time to mastering the technology and this time may not be immediately compensated for by timesavings made available by the technology. This will likely mean that some course topical coverage may need to be sacrificed.
- Technology is not 100% reliable. Instructors need to have backup and contingency plans. There must be means to provide technical support to both instructors and students. Furthermore, the more the technology is integrated into the course, the more susceptible the course is to significant problems when the technology fails.
- Students may not "buy in" to the technology and we waste both our time and their time.
- Students may over "buy in" to the technology and may not gain some of the key individual skills required by the subject material. This is especially likely when the students have complete access to the technology all the time such as with hand-held calculators or portable notebook computers.
- Instructors must typically spend additional time in course preparation to incorporate the technology into their lesson and take into account all the considerations made mention in this article (as well as those which I'm sure I've overlooked).

So how do we decide whether to use technology and, if so, at what level?

The primary driver to answering this question needs to be the benefit to the student. Will the technology aid the student in learning? Will it increase the student's efficiency and/or effectiveness in problem solving? Will the technology be of future benefit to the student?

Beyond these questions, there are practical concerns. Do I, the instructor, have the time and resources to properly incorporate the technology? Can I overcome potential "show-stoppers" with adequate contingency plans? Will the students be able to handle learning both the new technology and the course subject material?

Indeed, the incorporation of technology into the mathematics curriculum is not an easy challenge and it is one that will require continual exploration and experimentation. We can however learn from each other's experiences and more quickly attain higher levels of effectiveness by doing so. Hopefully this article has made some positive contribution along these lines.

How Technology Can Enhance Understanding in Multivariable Calculus

MAJ. Gerald C. Kobylski, USMA

The math software program we use in the Department of Mathematics at the United States Military Academy is Mathcad. Cadets see this program in three of their core courses, Discrete Dynamical Systems, Calculus I and Differential Equations, and Calculus II, Multivariable Calculus. When cadets begin our Multivariable Calculus course, they have already learned some of the fundamentals in Mathcad. In this article I will discuss how we utilize technology (Mathcad) in our course in order to enhance the cadets' understanding of the course's concepts.

The primary area where cadets utilize Mathcad is on two course projects. Goals for these projects relating to technology are to create a more in depth understanding of concepts taught in the classroom through visualization and to show the benefits of using technology in solving application problems. The first goal is accomplished primarily by graphing 3-D space curves and surfaces, the major topics for the first part of the course. The second goal is accomplished by giving the cadets more realistic problems that are difficult and time consuming to solve by hand, but straightforward using a computer.

Many students will not learn a math software program until the time when they absolutely must. In the two projects we gave last year, cadets took almost twice as long as what we planned for. A major reason for this was that they did not have a firm understanding of the software and thus spent more time learning it rather than exploring the mathematics in the project. The challenge instructors encounter is to prevent procrastination in learning the Mathcad skills. Additionally, instructors must decide how to best teach these skills given the time constraints in class.

Just as with other math software programs, Mathcad performs many of the operations that we study in the course. Some of these skills however, are too difficult to learn for the value of the understanding they provide the cadets. Other skills really do not add any understanding or benefit for a cadet. An example is calculating the dot product of two vectors. Although neat and easy, doing this in Mathcad does little to meet our project goals for technology.

We developed a set of 11 Mathcad skills that each cadet should know in the course and pinpointed for the cadets and instructors each lesson these skills applied. Instructors were highly encouraged to cover these in class and then were given a certain number of points to assess the cadets on these skills (usually on graded homework). Additionally, the cadets were told that at the end of the semester they would take an exam on the computer that tested their knowledge of the 11 skills. Having short homework exercises which focused on these skills helped the cadets become more confident with their Mathcad abilities. The homework also motivated them to learn the skills gradually during the course, rather than learning all of them the night before the project was due. Doing the above greatly focused instructors' teaching and student learning throughout the semester.

After teaching these 11 skills in two semesters, we decided that we could further narrow the skills down to five. These skills would still give the cadets a more in depth understanding of the Calculus concepts and show the benefits of using technology in solving application problems. Additionally, these skills do not require much teaching time. The first skill is graphing a 3-D space curve. Being able to see 3-D curves gives the student a better idea of the shapes of curves plotted over various domains. It also enables the student to more easily grasp concepts such as arc length, tangent vectors, velocity, intersection, and collision. The second skill is graphing a surface to include its level curves. Like graphing 3-D space curves, graphing surfaces gives the student a better idea of the shapes of surfaces plotted over various domains. Graphing surfaces also enables the student to more easily grasp concepts such as the location and classification of critical points, gradient vectors, directional derivatives, and Lagrange Multipliers.

The third and fourth skills that should be taught are finding derivatives (partial) and evaluating integrals (double). The fifth and final skill is solving systems of equations (linear and nonlinear). Performing these skills on the computer gives the student much more capability to solve realistic problems involving complicated derivatives and integrals and complex equations that either are difficult or time consuming to solve by hand.

Another benefit of using technology to solve problems is the ability to conduct a sensitivity analysis very quickly. Parameters in models can be changed in order to see if there is an affect on the solution. Such parameters may involve assumptions the student made. Repetitive calculations and "what-if" scenarios could also be quickly executed using a software program.

An obvious disadvantage to teaching computer skills is that it takes away precious time from the class and from the student's study time. No matter how much one tries to focus a student's learning, inevitably there will be some who "get lost." We find that most students who cannot easily graph the five skills mentioned above usually did not come away from their previous two Math courses with the level of competence in Mathcad they needed to.

Class time is precious. Student study time is also precious. We must constantly ask ourselves as teachers what is the most efficient way to teach our students what we want them to learn. Technology is a mountain that keeps getting bigger and bigger. We can certainly use it to our advantage in the classroom, particularly in teaching Multivariable Calculus. If we do utilize technology, we must be extremely careful that we do not overburden our students with learning the software. Being able to perform the five skills mentioned in this discussion will deepen the understanding of concepts taught in the course. Additionally, the time required to learn these skills should be minimal for our students.

How Technology Should Affect What and How We Teach

LTC. Joseph D. Myers, USMA

Technology should change the set of skills and knowledge that we have always thought of as fundamental.

Some skills are no longer important to most potential users of mathematics (e.g., most of our students) because technology is almost universally available that can execute them sufficiently well for us. These include techniques of integration, drawing and graphing, and root finding. Solutions to systems of equations, both linear and nonlinear, continue to be taught by hand but are better done with technology. The ability to make nearly effortless numerical evaluations has greatly reduced the importance of trigonometric identities, yet we still employ them just because they are available and we (teachers) know them. We often bemoan the fact that our students don't know the trig functions at special angles, but why do we value that so much? 30° and 60° angles are largely artificial; they show up so rarely in real applications that maybe we should satisfy ourselves with numerical evaluation as needed, just as we do for 31° and 59° angles. Cross products and curls are easily done with technology and there is absolutely no mathematical insight gained through calculating them by hand, yet we always teach and test them.

As we accept the fact that some skills such as those discussed above are no longer important, there is a corresponding set of technology skills that we need to be teaching. These include skills in numerical evaluation, computer algebra, linear algebra, numerical computation (of roots, eigenvalues and eigenvectors, combinations and permutations, and statistical measures), and graphing and visualization. We faculty have taken a very selective approach here; most of us teach a few random and personally convenient skills, but few of us have a comprehensive inventory of what students need or a plan for covering them all systematically. Reference [1] contains a sample plan of such skills, with representative realizations.

Some skills and knowledge remain important, and deserve added attention because of technology-induced atrophy. These include basic geometry, the algebra of polynomials, exponentials, and logarithms, the graphs, behaviors, derivatives, and integrals of the elementary functions, and the domains and ranges of the vector differential operators. Reference [2] contains a sample list of such skills.

Technology should allow students to see that, at least on one facet, mathematics is an experimental science.

In today's reform-minded environment we talk a lot about learning through student discovery, but don't always do a lot about it. We need to get serious by targeting places where deductive and analytic approaches have historically failed the students and see if technology offers a better approach. For example, deductive approaches to convergence of series are traditionally disasters for students, both in understanding and in execution. Instead we could replace these with a week of experiments designed to lead students toward a visceral understanding of convergence tests. For example: Guess at a test for convergence ($a_n > a_{n+1}$?). Given an intelligently selected collection of instructor-provided series, graph "enough" partial sums to tell if each is converging or diverging; can you use these to give evidence for or against your conjecture? Plot the ratio a_{k+1}/a_k; is there any relationship between convergence and the graph of this ratio over k? Does the harmonic series look convergent or divergent? Group its terms into packets such that the n^{th} packet contains 2^n-2^{n-1} terms and plot the sum of each packet; can you draw any conclusions? This approach is applicable to other traditional student problem areas, such as limits and continuity of multivariate functions.

Technology allows us to disprove conjectures in a straightforward manner, but we need to use it more to do "proof by example". Of course, what we really mean here is to do enough positive examples to convince the student that the conjecture is very feasible, and that (coupled with our assurance) he or she really should believe it. What this step also does is motivate the need for proof; with so many examples under our belt and the willingness to believe it's true, we're now motivated to pursue the proof (if we have an audience that has either the need or an inclination in that direction).

We should move beyond using technology to demonstrate toy problems and really start doing more involved problems

As technology proponents we have often touted the ability to do more realistic problems, but even with its almost universal presence, we usually don't do this. What we actually do is teach the traditional skills and techniques on the canonically easy problems, then repeat those problems using technology to show how it is done and how technology makes it easier. We shouldn't introduce nonlinear equations, uglier integrands, and more complicated functions just to show it is possible. Rather, we should actually take advantage of the power available by refining our models to more closely reflect reality and show how solving these refined models leads to refined solutions.

References

[1] Department of Mathematical Sciences, *Core Mathematics at USMA*, West Point, NY, 1999, pp. 28-35.

[2] Department of Mathematical Sciences, *Core Mathematics at USMA*, West Point, NY, 1999, pp. 24-27.

Technology in the Classrooms

LDCR. J. D. Skufca, USNA

The officer of today and tomorrow must be knowledgeable about technology, comfortable in using a computer, and capable of using these tools to improve mission effectiveness. To ensure that our graduates have these competencies, we are trying to integrate technology into the curriculum, and mathematics seems a logical place to start. We need to ensure that as we bring technology into the math classroom, we don't allow these tools to keep the student from learning essential skills. The overuse of calculators in the beginning math courses may be hurting our students' ability to master those skills.

Calculus for the Naval Officer is important. To perform well, our graduates need a technical background in the sciences (physics, electrical engineering, thermodynamics, etc.), and study of these topics requires understanding of the Calculus. Additionally, the systems that we use and the dynamic environment in which we operate the fleet require that the junior officer have an understanding of the "language of change." Most people reasonably infer that anyone who understands Calculus has other math skills as well: they would have basic problem solving skills, experience in implementing technical algorithms, a good understanding of trigonometry, and certainly (and perhaps most importantly) they would have a good grasp of the use of algebra. Computer algebra systems (CAS) and symbolic calculator provide an opportunity for a student to understand calculus without having those implicit skills.

Calculus is an important math skill for technical courses, but at a very basic level. Nearly all technical courses have some concepts that require the derivative or the integral to provide proper explanations. However, for our core courses at USNA, the degree of difficulty of these problems relatively low. Typically, students will deal with 2nd or 3rd degree polynomials, basic trig functions, and the exponential and logarithm function. Basic calculus with these simple functions must be understood to be able to understand the information in the core courses. The "by-hand" method of teaching Calculus I and II is sufficient to deliver these skills.

Algebra is essential in technical course. Every derivation requires some algebraic operations. If the student is not able to understand the algebra, the student will not be able to participate in the learning process of the actual derivation. The multitude of equations, therefore, are more likely to be grasped as separate ideas instead of related concepts, making the material more difficult to understand. Using the calculator to solve the algebraic operations will generally not be effective in aiding the learning process. The student achieves no greater insight simply because the calculator can manipulate one set of symbols to produce another. Furthermore, if the student must use his calculator to follow the derivation, then he is forced to disrupt one area of thought (i.e. physics) to consider another (rules of operation for a calculator). If the student can follow the algebraic operations without a calculator, the derivation never leaves the realm of being a physics problem.

Before the introduction of symbolic manipulation calculators, the core calculus courses developed both algebra and calculus skills because algebra was reinforced on every problem. A "difficult" problem usually involved difficult algebra and difficult calculus. The fact that someone understood Calculus did imply that they had the problem solving, algorithm recognition, trigonometry, and algebra skills required to operate in a technical military environment. Those supplementary skills, however, were not recognized for their individual importance because they were always present. In the day-to-day operation of the military, a junior officer will use algebra and trig skills much more frequently than calculus. In fact, most Commanding Officers would not expect that their officers remember much "Calculus," but they certainly expect them to be able to solve simple algebra and trigonometry problems. If they were able to learn Calculus but not algebra, there skills are not very valuable. It would be like a fighter pilot that knows how to dogfight, but is not able to take off and land. The fundamental skill (algebra), though more easily grasped, is essential.

Technology can do a lot of great things in the college mathematics classroom. The graphical presentation of concepts, especially with the ability to use animated displays to assist in explanation, cannot be equaled on a chalkboard. The system's ability to handle complex problems without error allows the class to study more realistic problems, which helps to show how the various concepts might be applied to real world problems. Reducing the amount of time that students and instructors must devote to the symbolic manipulation allows more time for investigating the concepts of Calculus. Giving every student the ability to graph any function allows complex expressions to be more easily understood. All of these things provide great opportunities for learning, and all have there place.

The use of technology, however, comes at a price. Our students come to us with good algebra and trig skills. However, those skills are not like the multiplication tables, firmly planted on the brain, never (almost) to be forgotten. The students are in the final stages of learning algebra. If we fail to exercise those skills, they will atrophy. Moreover, we will give the students the impression that once they move to the next higher course, the previous course becomes obsolete: remembering how to differentiate becomes obsolete after completing first semester calculus - the calculator will do it; integration - throw that away after plebe year - the TI-92 will give you the answer; this isn't high school - do the algebra on your calculator.

If students are allowed to use the calculator for symbolic calculations and solving equations, they will make this choice over handwork, not because it is easier, but because it eliminates errors. Because they have access to the calculators, we are no longer limited to problems that are solvable without calculators. Instead, we ask questions that would be difficult or impossible to answer without a computing device (or use of math tables). Because the student cannot easily determine whether a problem is "doable" by hand (without error), the calculator becomes the tool of choice. Many instructors believe that they emphasize the "by-hand" method and that they test this ability on quizzes and exams. However, the damage (loss of proficiency) has already been done because we do not force daily repetition of algebra skills. Testing for proficiency in "by-hand" calculation is generally such a small percentage of the test that a student with unsatisfactory basic computational and algebra skills may still be able to pass the course. Therefore, those students are allowed to pass through the system to physics, statics, dynamics, and EE without the skills they will need to understand the mathematical explanations that will be presented in those courses.

Basic math skills may not be the only things to suffer. Part of the reason that mathematics is considered a fundamental part of education is that it teaches a backbone structure of how to think logically and solve technical problems - logical reasoning. Problem solving is a process of identifying the basic characteristics of the problem, choosing the appropriate algorithm to solve the problem, verifying that the specific problem matches the algorithm, and implementing the algorithm. If the problem does not fit any known algorithm, that fact must be determined by the problem solver and a new algorithm developed (which we might call critical thinking). The process requires the solver to be able to recognize patterns to determine the appropriate algorithm. When working calculus problems by hand, the process of algorithm identification and implementation is exercised many times at many levels within one problem. However, if the problem is solved using a calculator or CAS, many levels of that process are removed, and only the highest level algorithms are required. The rest becomes merely a syntax issue. The student does not get the repetitive practice necessary to hone the general problem solving skills.

When our students pay two hundred dollars for calculators and two thousand dollars for a computer that can do all the things that we are asking them to learn, there is an inherent pressure to use the technology. However, in the introductory courses, this pressure should be opposed. The students must develop the fundamental understandings of calculus and they must become completely competent in the use of basic algebra. Without those skills, they will not have the necessary tools for success. The issue remains, however, that one must convince the students that they are "better off" doing by hand the same things that they know the calculator can do (without error). Luckily for us, most of our students have some background in athletics. They have spent many hours in practice doing some drill which cannot be directly applied to a competition situation. Perhaps that is theanalogy we need to draw. Math drills have been a key part of learning for a long time. Even though we are teaching at a much higher level, repetition remains a crucial part of the process.

Conclusion. The elementary college calculus courses serve as a foundation for further technical pursuits. To provide that foundation, these courses must reinforce the algebraic and basic calculus skills that will be required to understand the development of material within the technical courses. The calculator is counterproductive in development of these skills. Although the calculator makes the calculus more accessible in the short term, the overall effect on the technical curriculum is negative. Calculators should be reserved for higher level math courses, where the fundamentals have already been established and the fruits of the calculator can be enjoyed without the penalty of loss of basic skills. Where introductory courses desire to occasionally pursue the more difficult problems that warrant a CAS or calculator, it should be an adjunct to the fundamental principals beingtaught in the course.

Pros and Cons of Calculator Usage in the Classroom

LTC. Jeffrey S. Strickland, USMA

What are the pros and cons of using an electronic calculator in the mathematics classroom? There are too many to enumerate! However, there may be some principles that have been overlooked. Consider the notion that adults (us) use calculators to ‘get out of mathematics’ while students use calculators ‘to get into mathematics.’[1] The implication is that maybe we use calculators in a different way than our students’ use them. My experience shows that students’ experience with calculators in our classrooms will be fundamentally different from our experience of calculators.

Perhaps there is an underlying assumption that our students’ thought processes are essentially the same as our own. This is more of an attitude than a fact. There is a lot of empirical data in the literature that suggest otherwise. Many mathematicians believe that learning integration by parts is a rite of passage within the mathematical community and that calculators will harm the student’s potential to memorize the technique. This attitude is reflected across the mathematics curriculum.

> The school curriculum, however, is a conservative social institution. In this context, decisive change, even though based on logical argument and research, is likely to be resisted. Some aspects of school activity are treasured as fundamental: and proposals which appear to devalue these aspects encounter a backlash of personal and political prejudice and something called ‘common sense’. [2]

Certainly, there are some inappropriate ways to use the calculator. One inappropriate approach, which can be described as accommodating, is fitting the use of the calculator into an existing mathematics program. Research suggests that there are more powerful ways to use calculators; ways which use its full potential to promote learning. These more powerful approaches require that the calculator be fully integrated into the mathematics program. Still better, design the program around the use of the calculator.

Appropriate uses of the calculator release students from the tedious and mechanical constraints of calculation, enabling them to concentrate on meaning. Stanilas Dehaene states that, “The human brain behaves unlike any

[1] From a CAN workshop run by Angela Walsh and Hilary Shuard, Cambridge, April 1990.
[2] From Costello, John. (1993). The Precious Futility of Arithmetic. Mathematics in School. Vol 22, No. 3, p23.

computer that we currently know of."[3] Cognitive science suggest that it has not evolved for the purpose of formal calculation. This is why sophisticated arithmetic algorithms are so difficult for us to faithfully acquire and execute. There were times when the lack of technology required the acquisition of such algorithms. Those times may have passed.

Let's examine a military model. When mobile armor vehicles hit the stage, we didn't keep our horses just to retain the skill of integration by horseback. We can use the technology to enhance learning, just like we used armor to enhance fighting. Just remember that there are appropriate and inappropriate uses of armor as well.

If we haven't noticed yet, technology changes everything. Whether we like it or not, division and subtraction algorithms, as well as differentiation and integration algorithms, are endangered species quickly disappearing form our everyday lives–except in schools, where we still tolerate their quiet oppression.

Technology in Introductory Probability and Statistics

Dr. John C. Turner, USNA

For about 5 years, we have made extensive use of programmable calculators in our introductory probability and statistics course. For most of those 5 years, the course contained no statistics and was purely probability. We are beginning to add statistics, but only very small samples and simple inferences. Therefore, I will only address the probability aspect of the course.

We initially wrote programs to calculate the cumulative distribution function (CDF) for 4 basic random variables: binomial, hypergeometric, Poisson and normal. The three discrete distributions can be easily programmed using the formula for the probability mass function and summing to get the CDF. For the normal distribution, we used the approximation given in Abramowitz and Stegun. With the advent of the TI-92 and a bigger screen, the programs were modified so that instead of calculating the CDF, the programs calculated the probability of a specified range of values. The TI-92 has a button for infinity, which fit in well in the normal case. Infinity also makes sense in the Poisson case, but the program detects infinity and calculates the finite complement instead (and adjusts the answer, of course).

These programs have had several important impacts on the teaching of this course. Changing from the CDF to an arbitrary range of values virtually eliminated the confusion in calculating Prob(X>k). For discrete distributions, Prob(X>k) is generally not the same as Prob($X \geq k$). Since the CDF gives Prob($X \leq k$), there is some issue how to convert this into the desired probability. With the new version of the programs, the student asks for the probability that X is between 5 and N (inclusive) to find Prob($X \geq 5$). A simple programming change has eliminated a continuing source of frustration for students and faculty alike.

Over the years, we have noticed a much more important result of using the calculator programs. The speed of feedback seems to have a big effect on how well the students learn the material. In the "old" days, the student spent several minutes trying to enter the formulas into his/her calculator correctly. By the time an answer appeared, the student often had forgotten what the original question was. When the answer appears in only a few seconds, the student can stay more focused on what is being asked and how this calculation answers the question.

With this speed of feedback, problems became feasible that were not feasible before. Without a program, it was not feasible to ask Prob($X \leq k$) for, say, the binomial distribution, unless k was very small. The calculations were simply too time consuming. With the calculator programs, the students can do almost any value of k. Doing Prob($X \leq 50$) takes well under a minute. (On the TI-92, it takes about 5 seconds, in fact.)

"Guess and test" has become an important part of the course. Suppose we are looking for a 90% upper prediction bound for a binomial random variable. That is, 90% of the time, the random variable is less than or equal to what value? The student tries an initial guess. If the probability is too small, then the student should try a larger range of values (higher end point). If the probability is too large, then try a smaller interval. This would not be

[3] From Dehaene, Stanilas. (1997). The Number Sense. New York: The Oxford University Press, p. 134.

possible if each stage of the method required several minutes to compute.

"Guess and test" serves another important part in the course. It reinforces the relation between the probability being calculated and the quantity being varied. If value being tested produces too small a probability, should we try a larger value or a smaller value? In truth, the student doesn't need to answer this question. If a smaller value produces a better probability (closer to what is desired), then continue in this direction. If the new value is worse, then move in the opposite direction. But the instructor can use the opportunity to discuss why a given direction would be the proper one to pursue.

Lastly, I find that the greatest benefit of reducing the calculation time is that it gives more time to think about the problem. Most of the problems require anywhere from a few seconds of calculation (for a simple problem) to half a minute (for a "guess and test" problem). If we use an average of 15 seconds per problem, then I would ask for a full minute of thought before starting each problem. On a 50 minute exam, this means that up to 40 problems are feasible. This is much more than one would probably use, meaning that the student may have up to 2 minutes of thought per problem, or 8 times as much time thinking as computing. If we can achieve this ratio with our students, then we will have certainly accomplished something.

To encourage thinking about the problem, I try to begin each problem with a discussion of what we think the answer will be. When the desired answer is a probability, it usually suffices to narrow the answer down to: (1) near 0, (2) near 1 and (3) near 0.5. The reasoning that goes into this decision tells the student a lot about probability distributions. They will make a lot of use of the fact that roughly half the probability is on either side of the mean (but not exactly). For normal or near normal problems, they will learn the importance of the standard deviation. (Our normal distribution is not the standard normal. The student enters both the mean and standard deviation.) But regardless, it is another increase in the time spent thinking about the subject matter.

Finally, I encourage thinking about the answer after the calculation is done. "Is this answer consistent with what we expected?" "If we changed some part of the question, how would the answer change?" "Are there similar questions that have very different answers?"

In all aspects of life, one of the major impacts of technology is to increase the speed at which things happen. If uncontrolled, this means that bad things as well as good things will happen faster. This is true whether we are driving cars, flying planes or sending email. Therefore, considerable attention must be paid to ensuring that we limit the increase in bad results. The best way to do that is to spend a higher proportion of our time thinking and carefully considering what we are doing. This is good advice in both mathematics and everyday living.

Educational Games and Technology-Based Teaching Strategies

Dr. Frank C. Wilson, USAFA

Project Description

In an effort to provide students with a visual learning environment in and outside of class, thirty-six animated PowerPoint presentations and a series of Jeopardy-like computer games were developed in support of the Air Force Academy's differential calculus course.

The courseware development began in July 1998 and was first implemented in the 1998 Fall Semester in three 22-student sections. In Spring 1998, the presentations were revised and used course-wide in teaching 101 students. In Fall 1999, the presentations were again revised and used by 8 instructors teaching over 500 students. Instructors have repeatedly commented that the presentations make lesson preparation much easier.

The initial Jeopardy-like computer game was developed in August 1998 and was a big hit among faculty and students alike. Eight teachers customized the basic version to meet the needs of five courses. Over the course of three semesters, over 700 students have played the game. An enhanced version of the game was developed in September 1999 and used by 10 instructors teaching an additional 600 students. Sound effects and graphical animations were included in addition to the text-based questions.

Innovation and Effectiveness

Students have full access to the presentations and games via the local area network. As a result, they can revisit content covered in class at the click of a mouse button. This is especially helpful for athletes and others who are excused from class for a game or other higher priority event. They can review the material upon their return or download the presentations to take with them on the road. One student said that "PowerPoint displays and answers on the [network] drive really help me learn the material."

Many students find the presentations effective and the games fun. One student wrote "[The] PowerPoint and other computer programs were better than [any] I have seen. It really helped me to learn and kept me more interested." Because students are provided with handouts of the PowerPoint slides, they are able to focus in class on comprehension rather than regurgitation. A student commented, " [I enjoy] the excellent notes that you provide from the print outs of the PowerPoint presentations. I keep them in a folder and they are always useful for studying."

Significance and Transferability

The presentations marry the graphical genius of Mathematica with the presentation prowess of PowerPoint. Graphics are recolored and animated to create a stimulating learning environment. Concepts that are difficult to visualize in print are clarified with graphical animations.

Mathematica facilitated the creation of complex graphics and dramatically reduced development time. PowerPoint simplified the graphical animation process and provided a readily accessible presentation package. Exercises within the presentations stimulated the use of graphing calculators.

The presentations were designed to support the Calculus: Single Variable text by Hughes-Hallett, Gleason, et al. As such they are readily transferable to any one using the text. Customizing the presentation library to fit another text or to meet specific student needs is easily accomplished by anyone familiar with PowerPoint. The technique of importing Mathematica graphics and converting them to PowerPoint objects is detailed in Appendix A.

The Jeopardy-like games have been a hit among faculty and students. Eight instructors teaching five different courses have modified the initial game. Over 1300 students have benefited.

Enhancement of Student Understanding

Although all students benefit from the games and interactive learning aids, the target audience is students planning on pursuing a non-technical major. Many of these students experience high levels of math anxiety. By diverting their attention from their fear of mathematics to the external objective of winning a game in a group learning environment, they are able to more readily focus on learning the concepts. Additionally, the graphical learning environment in the presentations caters to the visual learner and helps them more easily comprehend mathematical concepts. As one student observed, "[the PowerPoint presentations] help me to visualize the math." The enhancement of student understanding is more than anecdotal: final exam scores in Spring 1999 in the areas of differentiation, precalculus, and calculus applications were 8.7% higher than Spring 1998 scores. As with any large course, there may have been other variables that contributed to the improvement. Despite the small sample size, student response to the use of the games and presentations was overwhelmingly positive.

Conclusion

This project has improved the learning of over 1300 students at the Air Force Academy and its impact continues to increase. Over 25 instructors have used the games or presentations to enhance their teaching. The employment of educational technology in the PowerPoint presentations and computer games has facilitated student mathematics learning and faculty lesson preparation. We will continue to refine and enhance these interactive teaching tools in the years to come. A student summed it up best when he wrote "I am learning a lot and comprehending well. The PowerPoint presentations are effective."

Appendix A: Converting Mathematica Graphs to PowerPoint Animations

1. Copy the graphic in Mathematica

Click on the graphic and press "Ctrl-C"

2. Paste the graphic in PowerPoint

Press "Ctrl-v" to paste the graphic in an open slide.

3. Size the graphic

Click on the handles and drag to the desired dimension.

4. Convert the graphic to a Microsoft Office drawing

Double-click the image and select "Yes" (This makes each element of the graph a modifiable object.)

5. If the font/line color of the graph is similar in color to the slide background, change the font and line color to a contrasting color

On the drawing toolbar, click the arrow next to the paintbrush icon and select a color. (This changes the line color.)

On the drawing toolbar, click the arrow next to "A" icon and select a color. (This changes the line color.)

4. Remove the white background of the graph.

Click on the slide background. (This deselects all selected objects.)

Click on the white space of the graph

Press the Delete Key

Repeat the above delete sequence until the white background has been removed (2 – 3 times).

5. Group the x-axis labels

Hold the Shift Key down and mouse click on each label

Click Draw on the drawing toolbar and select Group. (This groups the labels)

6. Group the y-axis labels

Hold the Shift Key down and mouse click on each label

Click Draw on the drawing toolbar and select Group. (This groups the labels)

7. Remove the line border from the x- and y-axis labels

Hold the Shift Key down click on an x-axis label and a y-axis label

On the drawing toolbar, click the arrow next to the paintbrush icon and scroll to No Line. (This makes the line invisible.)

8. If necessary, adjust the position of the x- and y-axis labels

Click on the appropriate label group

Use the arrow keys to adjust the label position

9. Increase the width of the x- and y- axis

Hold the Shift Key down click on the x-axis and the y-axis

On the drawing toolbar, click on the line thickness icon (three lines of different widths are on the face of the icon).

Scroll to 1½ pt (This changes the line width to 1½ pt)

10. Change the line width/color of the plot of the function

Click on the plot of the function

On the drawing toolbar, click on the line thickness icon (three lines of different widths are on the face of the icon).

Scroll to 3 pt (This changes the line width to 3 pt)

On the drawing toolbar, click the arrow next to the paintbrush icon and scroll to the desired line color. (This makes the line invisible.)

If you have multiple plots, repeat the above steps for each plot.

If desired, label plots with PowerPoint text box.

11. Animate PowerPoint objects

Right-click on the object you want to animate.

Scroll to Custom Animation

In the Custom Animation dialog box, click on the arrow next to No Effect.

Scroll down to the desired animation effect. (Wipe Right works well.)

Click the Preview button to see the effect.

Click the OK button when you're satisfied with the animation effect.