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EDITOR'S NOTE

Welcome to the 20th Anniversary of Mathematica Militaris! In June 1989, Chris Arney, Joe Arkin, and four managing editors from each of the service academies published the first edition of this bulletin with the purpose being, "to share information among the faculty and students of the mathematics departments of the four service academies." In this issue, we celebrate the history and traditions in teaching mathematics at these institutions. I hope you will find the articles as interesting and thought provoking as we have.

First, Tina Hartley and Fred Rickey review the history of using technology in the mathematics classroom. Their paper takes us on the journey from the first use of blackboards and string models to the modern laptop computers we now see in the classroom. This is followed by an article from Chris Arney on the changes he has noticed in the ten years between his retirement as department head and his return as a civilian professor at USMA. Next, Joe Myers shares his thoughts on how teaching at West Point has evolved over the past two decades along with some fond memories from teaching in the core mathematics program. Jim Rolf and Mike Brilleslyper (USAFA) then share their views on curriculum reform in the mathematics classroom, tracking the history of pedagogical trends and anticipating future developments. A lively rant from Brian Winkel, currently visiting at USAFA, encourages all of us to get on the Computer Algebra Systems bandwagon. Finally, David Spoerl updates us on the status of the operations research program at USNA.

In subsequent issues, we hope to include more information from and for students. Hopefully as you read this issue you will be inspired to submit some ideas of your own and discuss the possibility of submitting an article with some of your students. As always we welcome your reflections, new ideas, and challenges in mathematics and teaching in the academy environment. Be on the lookout for the next issue, coming this spring, which will focus on innovation in the classroom!

Classroom Technology at West Point

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Introduction

FROM its inception, the United States Military Academy at West Point has used technologies to enhance the learning experience. Frequently these technologies have had a profound and lasting effect on how mathematics was taught. West Point faculty and graduates greatly influenced science, mathematics, and engineering education in the early years of the United States. We will explore some of the most significant uses of technology in the classrooms at West Point, from the early adoption of the blackboard to modern day computers and software.

The Blackboard

"Professor Baron furnished me with Dr. Hutton's Mathematics, and gave me a specimen of his mode of teaching at the blackboard in the academy."[1] So wrote Joseph Swift, the first graduate of West Point, about George Baron, the zeroeth professor of mathematics (so called because he taught mathematics at West Point before the academy was founded in 1802). This is one of the earliest uses of the blackboard in the United States.[2] The next recorded use of the blackboard at USMA was in 1817 by Claudius Crozet.

Andrew Ellicott, professor of mathematics from 1813 to 1820, was famous for the perfect geometrical constructions that he made at the blackboard with cord and straightedge. He even had a small slate and sponge attached to his buttonhole so that he could do a computation at any time. John H. B. Latrobe, who entered the academy in 1818, later wrote "I am not sure that we had desks, but rather think that we were seated on benches against the wall, with a blackboard to supply the place of pen and ink and slates." He had another recollection of Ellicott: "while learning surveying, we were chaining a line from a point in front of his house to an angle of Fort Clinton, and back again. Our accuracy quite astonished the good old professor, to whom we did not admit that it was owing to

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[1] *The Memoirs of Gen. Joseph Gardner Swift, LL. D., U. S. A., first graduate of the United States Military Academy at West Point, Chief Engineer U.S.A. from 1812 to 1818*, p. 27. Privately printed, 1890. Available on-line at http://digital-library.usma.edu/libmedia/archives/swift/PART0001.pdf .

[2] Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts, *Tools of American Mathematics Teaching, 1800--2000*, Johns Hopkins University Press, 2008. Chapter 2 deals with the blackboard; see especially p. 23. The first mention of the blackboard was at Rutgers in 1779. Before that individual slates were used. The blackboard was used in a few schools in the US before it was used at USMA. See Charnel Anderson, *Technology in American Education, 1650-1900*, published by the US Dept of Health, Education, and Welfare, 1961.

our having used the same holes that the pins had made in going and returning."[3] Thus we see that there were two technologies employed in the mathematics department at West Point from the earliest days: the blackboard and surveying instruments. The use of the blackboard continues to this day but surveying was transferred to the Department of Civil and Military Engineering in 1930.[4]

The *Annual Report of the Superintendent of the United States Military Academy, 1896* contains a long (pp. 38-101) history of the mathematics curriculum by department head Edgar W. Bass. A "Description of the Section Room" has detailed information about the blackboards:

> "Upon the walls in oak frames, their surfaces flush with the face of the frames, are twelve or fourteen slates, usually 4 feet by 3 feet 6 inches. . . . They are all known by the generic name of blackboards. From the lower part of each frame projects a shallow chalk tray, having at its bottom still shallower drawers, and above each drawer a galvanized wire grating. The chalk crayons and erasers, when not in use, are kept on the grating in the tray, while the dust which these implements always generate falls into racks to support rulers and pointers." (p. 75)

Perhaps no one method has so influenced the quality of the instruction of the cadets as the blackboard recitations. Superintendent Thayer insisted on this form in the 1820s. Today it remains the prominent feature of academic instruction and the command, "Take boards!" is as feared today as it was in Thayer's day.

Olivier String Models

Descriptive geometry, which has evolved into computer aided design today, was introduced at West Point in 1817 by Claudius Crozet, who had studied it at the École Polytechnique. Crozet wrote the first English language textbook on the subject (1821) and this was followed by books by Professors Charles Davies (1826) and Albert Church (1864). All of these were used as textbooks at West Point, but the only technology used, besides blackboard, straightedge, and string for drawing circles, was created by cadets. They discovered that a drawing could be traced by putting a pane of glass on a washstand with a lighted candle beneath it. A completed drawing was placed on the glass and then the paper for the copy on top. Curiously, such behavior was not then considered cheating.[5]

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[3] Florian Cajori, *The Teaching and History of Mathematics in the United States*, Washington, Government Printing Office, 1890, p. 115.

[4] The 1930 Superintendent's Report, p. 3. These reports are available at http://www.library.usma.edu/index.cfm?TabID=3&LinkCategoryID=23 .

[5] The 1896 Superintendent's Report, p. 61.

On November 4, 1857, Professor Church sent Superintendent Delafield a list of models he wanted to purchase from Monsieur Fabre de Lagrange of Paris at a cost of approximately $450. Two days later Delafield placed an order for twenty-six models, "illustrating problems of descriptive geometry." Twenty-four of these marvelous, and now priceless, string models survive, but sadly we have little evidence that they were ever used in the classroom. Arthur Hardy, an 1869 graduate who later taught mathematics at Dartmouth, wrote that "In descriptive geometry the academy had a magnificent collection of models, but they were shown to us after the study was finished – in other words, mental discipline was the object – practical helps and aims were secondary."[6] The department took a different view, claiming the models "are of marked value to the members of the third class when studying descriptive geometry. Those showing the forms and methods of generation of certain warped surfaces seem to be of especial assistance to them."[7]

The Slide Rule

The first slide rule designed, manufactured, and sold in the United States was Palmer's Pocket Scale which went on sale in 1844. About the same time former West Point faculty member Ferdinand Hassler, and first superintendent of the United States Coast Survey, used a home-made slide rule.[8]

Currently, our earliest reference to teaching the slide rule at West Point dates from 1905:

> "For the purpose of acquainting the cadets with the various mechanical devices used as aids in performing calculations they have received instruction in the use of the slide rule and of several calculating machines. The required use of the slide rule in the solution of the problems in connection with the daily lessons had given to a large part of the class a satisfactory facility in the use of this instrument. An arithmometer,[9] one of the best of the calculating machines, has been purchased for the use of the department [of Ordnance and Gunnery]. This machine, together with other machines of like nature in the possession of the other academic departments, has been a subject of instruction."[10]

While this shows that the slide rule was used at West Point as early as 1905, it almost certainly had been used earlier, for by then half of the engineering schools in the United

[6] Quoted on p. 52 of *The Best School in the World,* by James L. Morrison, Jr. For information on the models, see Amy Shell-Gellasch, "The Olivier string models at West Point," *Rittenhouse*, 17 (2003), 71-84. Pictures of these models are available at http://www.dean.usma.edu/math/people/rickey/dms/OlivierModels.html .

[7] The 1896 Superintendent's Report, p. 78.

[8] Florian Cajori, *History of the Logarithmic Slide Rule* (1909), items number 25 and 21. For a picture of Hassler's rule, see http://museum.nist.gov/object.asp?ObjID=5 .

[9] Our note: Surviving in the department is a Brunsviga Midget System Trinks, serial number 71690. This device was patented 11 January 1910.

[10] The 1905 Superintendent's Report, p. 41.

States used slide rules.[11] Surviving in the department is a Thatcher cylindrical slide rule produced by Keuffel & Esser model 2795 with serial number 4013. The accompanying manual, *Directions for Using Thacher's Calculating Instrument* is by Edwin Thacher and is dated 1910. The department used to have an earlier model, 1741, serial number 589, which dates from the period 1892-1900, but it was transferred to the Smithsonian in 1958.[12]

Professor Echols, after clashing with Superintendent Mills over the number of failures in plebe mathematics, was sent to Europe in 1905-1906 to study the educational systems there. Upon his return he wrote,

> "It seems advisable that time be found in the course in mathematics to resume the instruction in the theory and use of the slide-rule which was once taught in conjunction with the course in surveying and is now taken up in the course of Ordnance and Gunnery. It could be best added to the course in trigonometry of the 4th Class year, its use to continue throughout the next three years."[13]

This shows that the slide rule was taught at West Point before 1905, but, thus far, documentation is lacking.[14] We anticipate finding additional documentation.

Instruction on the use of the slide rule continued at USMA, with various departments involved. In 1914, the Department of Drawing prepared a slide rule manual, and in 1930 the Department of Physics did the same. In 1944 the Mathematics Department resumed the teaching of the slide rule.[15] They used Keuffel and Esser slide rules and a manual by Naval Academy faculty Lyman M. Kells, Willis F. Kern and James B. Bland.[16] The Department also had 27 eight-foot rules for demonstration purposes, one in each classroom.[17] The amount of time devoted to the slide rule in plebe mathematics varied over the years from 5 to 9 classes (to a maximum of 12 hours), and then the slide rule was used by other departments in later courses.

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[11] Charles A. Holden, "The use of calculating machines," *Engineering News and American Railway Journal*, vol. 45 (1901), p. 405. Holden, an instructor at the Thayer School of Civil Engineering at Dartmouth, received responses from 24 engineering schools. The students at exactly half of them used the Mannheim Slide rule 'much' or 'constantly.'

[12] Email communication from Peggy Kidwell of the Smithsonian, 6 November 2009.

[13] Echols, *Report of Visits to Foreign Schools and Recommendations Resulting*. Notes on the report of Echols at http://www.dean.usma.edu/math/people/rickey/dms/DeptHeads/Echols-Europe.htm .

[14] The slide rule was used at the Naval Academy from 1929 to 1976 [T. J. Benac, "A Brief History of the Department of Mathematics," http://www.usna.edu/MathDept/website/mathdept_history.pdf]. At the Air Force Academy it was used from 1955 to 1974 [Frederick V. Malmstrom, "When slide rules ruled the rockies," *Checkpoints*, December 2008.]

[15] The 1943 Superintendent's Report, p. 3.

[16] Mathematical Department Diary, 1948-49, 1955-56, 1957-58, 1959-60, among others.

[17] One of these classroom Log Log Duplex Decitrig sliderules is pictured in the 1957 yearbook, *The Howitzer*. The caption is cadet humor: "Jim Henthorne proves to Captain Genebach that 2 and 2 are 5. Keuffel and Esser please note."

In a "Report of Activities, Training Aids Committee" dated 19 June 1961 there is mention of a visit from a K & E representative "regarding plastic slide rule for use with the vu-graph," a type of overhead projector. In February, the representative Mr. Thomas "arrived with a pilot model of a plastic slide rule for demonstration and comment." The department made seven suggestions for improving the model and it was ready for use in the fall of 1962. Only one of these plastic slide rules for use with the overhead survives, for they had a tendency to soften and twist when left on the overhead and were then unusable.[18]

The 1961 *Howitzer*, the cadet yearbook, has a photo of a cadet who has been stabbed in the back by his slide rule.[19] The slide rule itself was killed by the calculator.

The Overhead Projector

In the 1940s the Army realized that equitation was not a skill that Army officers needed, so the riding hall fell dormant, becoming an enclosed parking lot. The Mathematics Department head, Colonel Bessell, realized that the building could be converted into a classroom building. As chairman of the Academic Building Committee:

> "General Bessell was largely responsible that Thayer hall is one of the most modern and completely equipped classroom buildings on any campus. He insured that the latest advances in teaching technology were considered for its numerous classrooms, auditoriums, laboratories and museum."[20]

When Thayer Hall was created out of the riding arena in 1959, the classrooms were designed with blackboards all around the room, thus perpetuating the infamous command "Take boards!"

Bessell was cognizant of the importance and value of using audio-visual aids to improve teaching. He saw to it that each mathematics classroom had an overhead projector. He was also able to procure mechanical computers to support the instruction in his department.

The earliest overhead projectors date from the 1870s, but their first widespread use was in bowling alleys in the 1930s. During World War II, the armed forces used film strips and overhead projectors to train troops. This technique was adopted because there was a shortage of trainers. It was not until after the war that overheads became common in U.S.

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[18] Mathematics Department Diary, 1960-1961, tab 13. For additional information about these overhead slide rules and the use of the slide rule in education, see Chapter 4 of Kidwell et al., cited in footnote 2. The overhead rule was designed so that both sides of the duplex slide rule could be seen simultaneously.

[19] We would like to think that this was a bilingual pun, but doubt many cadets knew the German word "Stabrechnen."

[20] General Orders Number 54, 12 May 1965, Retirement. He is referred to as 'General Bessell' for by this time he was Dean of the Academic Board.

classrooms. It is not surprising that some of those first classrooms would be at West Point.[21]

The Mainframe Computer

In the spring of 1959, the department offered a "machine calculator seminar" that was completed by 131 Third Class students. It was so successful that it was offered again the next year. The seminar consisted of four lessons in the evening that cadets could volunteer to take. The machines used were the Monroe LA-160X, Marchant ACT 10M, Friden ST-10, and Monroe CAA-10. Detailed instructions were provided for how to add, subtract, multiply, divide, extract square roots by the divide and average iteration, and to do correlations (least squares).[22]

During his tenure as Dean of the Academic Board, Brigadier General William Bessell, former head of the Department of Mathematical Sciences, was instrumental in establishing the first computer center at West Point.[23] Along with the Heads of the Department of Military Engineering and the Department of Electricity, he established an Academic Computer Committee to set goals and recommendations for the use of computers at West Point. The committee concluded that exposure to computers was a necessary component of the cadet academic program, and decided on a central Academic Computing Center as the most economic way to provide computing power to cadets. The Academic Computing Center was thus established and opened in December 1962, with a General Electric 225 digital mainframe providing the computing power.[24]

The Mathematics Department initially expressed reluctance to include computer instruction in their core courses, due to time constraints and a concern that the new technology would not survive. This reluctance was quickly overcome and a computing problem was assigned to the entire fourth class in the Academic Year 1962-63 as part of their plebe math course. Two months after the Center had opened, every cadet in the Class of 1966 had written a program and run it on the GE225.[25] The purpose of the

[21] For a comprehensive history, see "The overhead projector: snapping the class to attention," pp. 53-68 in Kidwell et al., cited in footnote 2. A picture of an overhead in a 1961 USMA classroom is included.

[22] Mathematics Department Diary, 1958-1959, Tab 4. Mathematics Department Diary, 1960-1961, tab 15; Memo of 27 January 1961. "Seminar in desk-computer technique," *Assembly*, vol. 19, no. 3, Fall 1960, p. 20.

[23] There are probably many things that motivated Bessell to introduce computers. For example, he attended the May 1952 meeting of the NY MAA section where William H. Durfee of the National Bureau of Standards gave a talk on "High speed computing and its effect on teaching" wherein he argued that some numerical analysis could be introduced into regular calculus courses [*American Mathematical Monthly*, 59, 587-590].

[24] Kenneth L. Alford, Gregory J. Conti, David B. Cushen, Eugene K. Ressler, Jr., William Turmel, Jr., and Donald J. Welsh, "Computing at West Point, revolution to purposeful evolution," pp. 561-585 in *West Point; Two Centuries and Beyond*, McWhiney Foundation Press, McMurry University, Abilene, Texas, 2004, edited by Lance Betros.

[25] *Visitors Guide to the Academic Computer Center and its Historic Displays*, p. 24. This brochure contains a picture of the Academic Computer Center in room 104, Thayer Hall.

assignment was to compute to seven digits of accuracy the natural logarithm of the numbers 251 through 360. One solution to the problem, submitted by Cadet Torrence M. Wilson (USMA 1966),[26] consists of a stack of 33 cards (labeled “West Point Basic Programming System Instruction Cards”), with bubbles filled in with pencil to indicate instructions to the computer. The program started with a given value of the natural logarithm of 250, and used that to compute the value for the next integer using an estimation formula:

$$\log x = \frac{170.5}{1024}\left(\frac{1}{\text{Old x}} + \frac{1}{\text{New x}}\right) + \frac{1366}{1024}\left(\frac{1}{\text{Old x} + \text{New x}}\right).$$

The flowchart included with the solution indicated that the program would then loop until the desired degree of accuracy was reached. In an attachment entitled “Hints for Programming,” apparently a document from the Math Department, the student is advised that “Proper application of [the equation above] in programming will minimize the amount of looping and conserve machine time.”

Although the original purpose of the Academic Computing Center was to provide computing power for cadets, eventually it was used to handle many administrative tasks as well. It is hard to imagine today how burdensome some tasks were before the computer was widely available. For example, the Academy has a long tradition of posting student grades weekly in the sally ports. Determining cadet averages was such an onerous process that in January 1941, the Department of Mathematics, “with the assistance of the Departments of Civil and Military Engineering, Chemistry and Electricity, Economics, Government and History, and Physics,” produced a *Tables of Averages* in two volumes to assist in this task. The first volume had the Tables 11 to 110 inclusive and the second had Tables 111 to 150. The number of the table corresponded to the number of lessons. For example, if the cadet had a total grade of 49.7 in 18 lessons, the average grade would be 49.7 divided by 18. By looking at Table 18, one could find the average without performing the calculation by hand. The tables bear the annotation “U.S.M.A. – 1-31-41 – 180” on the cover, so 180 copies were printed. This large number indicates that they really were a benefit to the faculty.

The Electronic Calculator

On 30 October 1974, the Applied Science and Engineering Committee and the Basic Sciences Committee recommended that cadets be allowed to use “electronic calculators” beginning 1 January 1975. Aware of equity issues, calculators could not be used by members of the class of 1975 on graded work unless every cadet in the course had one. Members of the other three classes were required to purchase “suitable calculators” from the cadet store by “the beginning of academic studies in September 1975. . . . From that

[26] Wilson retired from the Army in 1987 as a Colonel. He is currently a physician at the Mayo Clinic in Rochester, Minnesota. The cadet work is in the Departmental Files in a folder marked “Historical Item . . . first computer program submitted by a cadet.”

time forward the calculator will replace the slide rule as the principal computing device employed by cadets, and cadets will not be required to own a slide rule." The primary reason for introducing calculators was "to increase the effectiveness of classroom instruction, particularly in the mathematics-science-engineering subjects." The committee precisely enumerated the capabilities that calculators must have: These included basic arithmetic, trigonometric, logarithmic and exponential functions with eight or more digits displayed in scientific notation and rechargeable batteries.

The committee did not specify a brand or model of calculator, but there was only one that fit the specifications, the Texas Instruments model SR-50 which was announced on 15 January 1974 and sold through the mail at $169.95. The first class to be issued calculators, in the summer before they began classes, was the class of 1979, which was issued the TI SR-50. This action of the Academic Board was applauded by General W. E. DuPuy who wrote to Superintendent Berry: "I am pleased to note your introduction of electronic calculators at the Academy. This forward looking action is directly in line with the widespread use of pocket calculators throughout the TRADOC school system.[27]

We cannot discuss each of the calculators used but note, for the record, the calculators used and the first classes which were issued them: TI-58C (1985), TI-59 (1986), HP-15C (1987), HP-28S (1994), HP-48SX (1997), HP-48GX (1998), TI-89(2003).

In the fall of 1999, plebes were issued the TI-89, a graphing calculator with a computer algebra system. The previous class used the HP48G graphing calculator. Faculty member Mary Ann Connors, who has served as a consultant for Texas Instruments since 1994, wrote an 83 page pamphlet, which was distributed to cadets, on using the TI-89. She and LTC Kathleen G. Snook (USMA 1980; the first class to graduate women) studied the impact of these two calculators on student performance and found the TI-89 to be superior.[28]

Computers and Software

The Academic Computing Center continued to be the only computing resource for cadets for some time. However, with the rapid rise of the personal computer, the Academic Computing Committee decided in 1984 that each cadet should have their own personal computer. Thus in the summer of 1986, the class of 1990 was issued computers at the beginning of the academic year. The computer chosen was the IBM Zenith 286, with a dual-floppy disk system and 1/2 megabyte memory, and the use of computers as part of the core math curriculum really began to take off.

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[27] *Proceedings of The Academic Board, USMA*, No. 87, 1974, Tab EE. *Assembly*, March 1975, p. 19. http://www.vcalc.net/ti-hist.htm . Dupuy to Berry, 1 August 1975; *School Study Files. Curriculum Study*, file 1011-04.

[28] Connors, *The TI-89 in Discrete Dynamical Systems and Calculus*, 1999. Connors and Snook, "A technology tale: integrating hand-held CAS into a mathematics curriculum," *Teaching Mathematics and its Applications*, 20 (4), 2001, pp. 171-190.

From this time through the early 1990's, each core math course included as a course objective the need to “develop cadets' abilities to employ calculators and computers as analytical aids.” In 1993, the Core Mathematics at USMA pamphlet lists the topic of Scientific Computing as a Mathematical Thread. The Mathematical Threads were designed to outline areas of expected student growth throughout the core math program. The objectives of the Scientific Computing thread were:

- Appreciate the role of machines as aids in learning and doing
- Manipulate and analyze data
- Interrelate symbolic, numerical, and graphical representations
- Recognize the capabilities and limitations of computational aids
- Perform simple programming

These objectives remain largely unchanged today.

Derive was the first computer algebra system used in the core math program. It was used for plotting, symbolically calculating derivatives and integrals, computing limits, and other tasks. How it came to be used is an interesting tale. At the annual mathematics meetings in January 1989, department head Colonel Frank Giordano was talking to Professor Marvin Brubaker from Messiah College. In the course of their conversation, Brubaker noted that someone from the Naval Academy was talking with the people at the Derive booth. When the Navy professor moved on, Brubaker introduced Giordano to the Derive people and before long, Giordano saw the great educational benefit of having Derive available to all cadets on their personal computers, and he arranged to buy copies at a good price. Back at West Point, when the lawyers found out, they vetoed this agreement because the contract had not been bid. So a call for bids was put out, but the creator of Derive would not underbid his distributors, so the cadets paid a much higher price.

In 1996, Mathcad replaced Derive as the primary software used in the core math program. Other software programs used simultaneously during the 1990's include QuattroPro, Microsoft Excel, and Minitab. In 2002, Mathematica replaced Mathcad, and is still in use today.

The Core Mathematics pamphlet had always listed a set of “Mathematical Recall Knowledge,” a basic list of skills and ideas in which the cadet was expected to be proficient upon completion of the core math program. Beginning in academic year 1995-96, the pamphlet also lists “Required Skills in Scientific Computing.” The skills are listed for both the calculator and the various software programs in use, and include such skills as “Solve nonlinear systems of equations” and “Approximate definite integrals numerically.”

Throughout the 1990s, the Math Department maintained several classroom computer labs, which instructors would use periodically during the semester to teach cadets how to perform these required skills on the computer. Teaching this technology remained difficult, however, as it was not frequently reinforced in the classroom, and students could not easily replicate the computer lab work on their own computers. In the fall of 1999, the Math Department, in conjunction with the Economics Department, conducted an experiment to determine if cadets should be issued laptops instead of desktop computers. Laptop computers were issued to 32 cadets in the class of 2002, who were then grouped into separate sections for their Multivariable Calculus and Economics courses. Instructors in these sections experimented with different ways of integrating the new technology into the classroom, and evaluated the results through exam performance and instructor and student feedback. The conclusions drawn from the experiment were that students' basic skills did not suffer as a result of increased use of technology, while their ability to visualize and understand problems improved through the frequent use of computers in the classroom.[29] Based in part on the results of this experiment, three years later the class of 2006 was the first class to be issued laptops for their personal computers. The laptop issued was a Dell C840 with a Pentium IV processor and 256 megabyte main memory.

Conclusion

West Point has frequently been on the forefront of technological innovation in the classroom, and the adaptation of new technologies from the blackboard to powerful personal computers has shaped both the methods of teaching and the material that is taught. While there is still debate in the mathematics community about the use of modern technology in the classroom, it does seem clear through these historical examples that appropriately using these tools greatly contributes to student learning. Rather than penciling in 33 programming instruction cards to compute a logarithm, students can now focus on the concepts and ideas and thereby gain a greater understanding of mathematics.

Play it Again, Sam – a 60-year Look at USMA (With a Math Focus)

Dr. Chris Arney
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THIS semester I am back teaching plebe cadets at USMA much like I did 30 years ago in a course much like the one I helped develop some 20 years ago in a department I left almost 10 years ago. I won’t even mention that I personally studied plebe math 40 years ago in a world that I entered 60 years ago. Obviously, I am thrilled at the opportunity – most people don’t get to relive the best parts of their life. Given my circumstance, I do have people occasionally ask, “What has changed? Are things better or worse? What about cadets these days?” I will address these questions here and try to tell you how I think the

[29] Jim Glackin and Joe Myers, “Laptops in the Classroom”, 1999. Unpublished manuscript.

eras compare. Before I begin, let me give a warning – these comparisons are very difficult to make. No one except Fr. Gabe Costa in the USMA Math Dept is 100% sure Babe Ruth is a better power hitter than Hank Aaron or Barry Bonds or A Rod. Even when you have data, comparisons over time (when everything in the world, including yourself, has changed) are just plain difficult. But we are so tempted to understand change that we continually make these comparisons, and we mostly see ourselves and "the good old days" in idealistic, rose-colored light. Another warning about what is to follow: there is plenty of free advice on all sorts of topics, so stop here if you don't want any. But if you choose to continue, then my two cents and free advice follow.

Cadets seem to be unchanged --- some energized, some not; some insightful, some not; some remember high school mathematics, some have forgotten; some work hard, some don't. They all want to serve their country and help change (or rather many want to *save)* the world, and in many of these regards, they appear to be a very homogeneous group. They do differ in some important ways, however. Unfortunately some cadets obviously think of math as a body of facts and recipes to memorize and regurgitate. For them, abstract, complex, college-level math is very difficult – they have reached memory capacity and they either can't memorize much more or they have forgotten too many fundamental skills to move forward. They have no profound understanding or intuition to rely on. To them, math is like a Saturday afternoon TV movie – something to experience, but with no lasting residue to empower their world as thinkers and problem solvers. In my view these cadets, even those that somehow pass and do well in their courses, will never be good scientists (physical, social, behavioral, life, mathematical, or military) or engineers. They will have to use other (perhaps more humanistic, non-quantitative) skills to succeed in life. But the good news is that many can and will do just that because they are amazingly bright in ways of thinking that give them an appreciation for a variety of cultural perspectives of the postmodern world. Today, I see many more cadets skilled in these perspectives than I remember seeing in the past. Nevertheless, there still are many cadets who can and do see the power of mathematics as the language of science, engineering, and problem solving. Those cadets are able to build more and more mathematical knowledge and insight that allows them to model the real world, to inquire and analyze phenomena, and to understand complexity. These cadets believe they can – and they probably will – change the world, if not save it. And amazingly there seem to be more cadets like this than ever before too. It's the number of in-between cadets that seems to have shrunk. In my opinion, there are many more cadets today than ever before who possess special talents and skills that give them potential, power, passion and purpose. The variance of their interests and skills is greater than ever, even if the mean seems unchanged. No longer is every cadet cut from the technology/science/engineering mold. Academic diversity is alive and growing at USMA. So I change my vote --- **Cadets have changed for the better *and maybe some* for the worse**.

Faculty members seem to be the same --- Faculty members at USMA have always wanted the best for their students and have worked hard to give it to them. We definitely strive to make inefficient learning processes more efficient through streamlining. Unfortunately, I do worry that at times the result is that we end up doing things that are more like training than educating. In our zeal to give cadets all the opportunities to learn, we sometimes reduce their own investment and effort in learning. As Millennial students, they expect to be guided, dazzled, entertained and successful. Obviously, they will need to mature in many ways to become life-long learners, capable of solving complex problems long after the instructor is gone, and the real world demands their independent mental engagement. Back when I was a cadet and during my time as a new instructor, the "old corps" faculty demanded cadet attention and engagement. Today's faculty earns these through their efforts to inspire the cadets. This new way is longer-lasting and more meaningful. So I guess I'd better change my vote again **– the faculty has changed** (and that's a good thing.) Today's cadets would completely frustrate the old faculty and their teaching style would not meet the needs of the modern student.

The world has changed --- I served for 30 years in the military and fortunately never served in combat. It may be that no one in the Army will be able to say something similar for quite some time. My world was relatively peaceful and simple – superpowers threatened each other and eventually compromised, technology advanced, we coveted the next fancy gizmo, houses and malls became bigger and more grandiose, TV added more channels, and each succeeding generation was more affluent and more educated than previous ones. The world was one big competition focused on growing, improving and winning. In my view, none of those patterns hold true anymore. In particular, we worry much more about the future of the world and now our lives are dominated by pop culture, terrorism, a search for security, and a desire for instant gratification with lots of social networking thrown in. We no longer believe that more technology or more education is always the answer to problems of the day – we view everything with skepticism and all truths are thought to be relative to one's own perspective. We have moved from the Age of Reason (and technology) to a postmodern world of doubt, worry, relativism, a lack of faith and confidence in science and traditional value systems/philosophies, and (fortunately) a world with increased connectedness, participation and cooperation. I have always believed that cooperation and participation produce better results than approaches emphasizing competition and winning, so I like these latter trends of the modern world despite all the anxiety that comes with it. Thus, I disagree with the views of many cadets who think the world needs saving. It is a good place and is getting better. Obviously, the world **will continue to change** (sometimes led by USMA graduates) as it always has – in ways that will be both good and bad, and (as always) wholly unpredictable**.**

The Army is changing --- and that almost certainly will continue for a good long while. The Army has taken on new and expanded missions – humanitarian assistance, disaster relief, peace-keeping, nation-building, anti-terrorism, counter-terrorism, security operations, joint warfare, and for old-time sake, large-scale combat operations. This role expansion means that cultural awareness, interdisciplinary problem solving, and human caring and cooperation are essential skills needed by future Army leaders if they are to be successful in completing their missions. The buzzwords PMESII (Political, Military, Economic, Social, Infrastructure and Information systems) and DIME (Diplomatic, Informational, Military and/or Economic) are key concepts in many areas of irregular warfare and effects-based operations that the Army is encountering. Sophisticated technologies will give us robots and machines that work hand-to-hand and mind-to-mind with our soldiers. As such, military leaders will need to understand all the complexities of those technical changes, along with the social and behavioral challenges they will pose in virtually all military organizations and operations. And, because of intense media interest and the increasingly personal nature of missions, junior soldiers (including recent West Point graduates) can and will affect the policies of the country. The Army's net-centric approach to modern warfare now requires officers to know information and network science principles, capabilities and their applications. As Academy graduates rise in rank and obtain leadership positions over the coming decades, the complexity of their mission to defend our country and to win peace in the world will increase dramatically. I hope that I am wrong in my perception that the present-day Army is preparing its future leaders to win the previous war (counter terrorism in Iraq and Afghanistan) rather than the next operational challenge our country will surely face. To wit, shouldn't our preparation for future leaders be more concerned with the burgeoning concerns and need for nation building in Africa (genocides, poverty, and pirates), China and the rest of Asia than it is with the current protracted conflicts? **Change is the one constant for our Army.**

The Academy has changed --- Of this, I am sure. Perhaps it is more like it was before I was a cadet in 1967. In some ways it seems like 1955 all over again. With the Army deployed in operations, the cadet culture has become focused more on military issues (as opposed to academic ones) than I have ever seen it before. Today's Spartan-like cadets are much more near-sighted, intense, and focused on military matters than when I left West Point in 2001. As a result of changes in faculty (more civilians and PhDs) and world events, the Academy had by the year 2000 established itself as a very academic-oriented institution, and in those days cadets took what I view as a more Athenian long-term, broader view of their educational experience. Now they are much more concerned about their first military assignment – they know what branch, assignment and unit they want when they enter the Academy. They long to deploy to the operational Army to lead a platoon and save the world. And the new military instruction appears to be a great way to prepare them for their future. They have military interests that in the previous era (1960-2000) were rare. The current cadet is probably much more like cadets of the 1940s and 1950s than of those in the more recent generations. Today's cadets want their courses and lessons to be relevant to their lieutenant time. Their interests are rooted in the idea that their experience at the Academy should make them better platoon leaders so they can save the world from its current turmoil.

The discipline of mathematics is changing from being predominately the language of science and engineering to the language of interdisciplinary problem solving and quantitative inquiry. No longer are physicists and chemists and engineers the sole or primary colleagues of the applied mathematician. Applied mathematics now plays significant roles in the social sciences, humanities, and arts. I spent the last five years making this mathematical transformation, or perhaps a cultural revolution, the emphasis of the Army Research Office. ARO's math division now focuses on research associated with interdisciplinary problems that often involve social, behavioral, informational, cognitive, or life sciences. Applied mathematics now is much more connected to the messy, complex, real world than it has ever been before. Today, applied mathematics is a powerful tool used by liberally educated people to produce change in the world via quantitative and qualitative analysis and high-powered computation.

The Curriculum has also changed (a little). Before I left 10 years ago, I had a hand in introducing IT (information technology) into the core curriculum (replacing some of the engineering science). I think it was a wise move because, as a result of these innovations, cadets can see the power of IT in their professional lives. But frankly, I am very surprised there have not been more changes. I think the world and the Army are substantially more focused on social and cooperative (interdisciplinary) concerns that produce a greater need for extensive consideration of cultural studies, social sciences, life sciences, and the liberal arts that address values-based, relevant-to-society perspectives. This approach leads to the formation of habits of the mind that, in turn, can lead to growth of broad knowledge and awareness of the wider, complex, real world (communication, problem solving, inquiry, analysis, modeling) that go beyond traditional mathematics. In this new world, math is the tool that helps future leaders (i.e., today's cadets) to perform quantitative and qualitative analysis of complex systems. My preference would be to create curricula that focus even more on liberal arts (meaning a bigger set of core multi-and inter-disciplinary values-based courses) that actually reduce the specialization requirements of the major, and a shift in pedagogy that involves much more hands-on problem-based learning, service learning, cooperative learning, and large-scale interdisciplinary case studies and projects. Graduate school and career schooling are the places and times for specialization and vocational education in their professional lives. As such, I hope USMA's **curriculum changes even more** to become increasingly interdisciplinary, liberal, values-based, and problem-focused.

I will never change. Of course, that is not correct. **I have changed** --- I walk slower and grade less often. And if you have read any of this, you know I am more of an idealist and optimist than ever. I am more patient and tolerant (especially of those cadets that do not enjoy math.) Yet, I love math even more than ever and believe it is a very important part of cadet education. I do think many of the cadets will do exactly what I hope they will do with their education – change the world, even if they do not need save it like the superheroes they aspire to be. Changing the world has always been the more noble, important, and sustaining goal, embedded in the mission of the academy, and more particularly, of the mathematics department – and I hope it continues to be so.

So, my bottom-line, dare I say, is this: USMA needs to become a post-modern Academy meaning it certainly and thankfully is neither Athens or Sparta, but rather, an uncharted entity and scholarly community, driven by important intellectual core-values, liberal education questions, new Army missions, complexity, diversity, emotion, multiculturalism, emerging technology, networking, chaos, and, above all, principles and values – a brave, new institution for a brave, new world.

A Few of My Favorite Things

Dr. Joseph Myers
Division of Mathematics
Army Research Office

WHEN Prof. Swim asked for thoughts on our Core Mathematics programs for this issue, I thought it might be good to take a simple approach. So taking a cue from SNL's Celebrity Jeopardy, I've chosen to respond within the category "Things I Like," and also to share a few related stories from USMA along the way.

Teaching small sections is very rewarding. The Department of the Army has committed to resourcing us at sufficiently high levels, on the order of 24 instructors for a 1300-student core course, such that nearly all academic sections contain 18 or fewer students. That means I have the opportunity to learn each student's face, to be able to greet them by name outside the classroom, to learn their strengths and weaknesses and motivations, and to work with and challenge them individually in class. Occasionally you hear an instructor reminding a student of the great opportunity the nation has given them as a student here and how they are therefore bound to take advantage of it. That reminder also applies to me as an instructor; the services' commitment to resource us at these levels compels me to make the most of my contact with cadets.

Some of my favorite students have been those who demonstrated an outstanding work ethic, or genuine inquisitiveness, or an undeniable determination to succeed, but without necessarily being very talented in our discipline. But I must admit that I especially admire, appreciate, and enjoy working with those top students who are talented in mathematics. Every year, instructors in our core program work with incredibly talented students such as Hertz and NSF Scholar Andrew Fedorchek '89, Hertz Scholar Marcia Geiger '92, Marshall Scholar Ray Eason '94, Rhodes Scholar Zac Miller '06, Rhodes Scholar Jason Crabtree '08, Rhodes Scholar Liz Betterbed '10, and many others. Every year, there are not just future national scholarship winners such as these sitting in our classes, but also many more very talented students who will attend top graduate schools in our and related disciplines. It is exciting to work with the talented plebes and yearlings in our core sections and to see them as the people who will carry on our love of the discipline.

We all appreciate the diversity at USMA. As a cadet in the mid-70's, my faculty was a uniform, and uniformed, bunch of 30 year old white males. One of our most important jobs as officers is to be expert team-builders, and one of our most important lessons-learned in team-building is that we are collectively smarter, more capable, and more robust when we incorporate a variety of experiences and points of view. What makes us more diverse is what makes us stronger. Now, as a faculty with more women, minorities, civilians, and senior officers in the classroom, we do a better job of learning from each other and inspiring and motivating our students. Of course, the other aspect of developing a diverse team is empowering individuals to contribute according to their strengths. We practice this in the value we place on innovation and the opportunity to practice it.

It may seem like a small thing, but we have spent a lot of energy over the years choosing the right textbooks for our core programs. I found very early on that I appreciate texts which are more complete rather than less. I benefit from having it all: good topical coverage, theoretical development, statement of applicable theorems or conditions, a few demonstrative computations, interesting examples and applications, interesting problems which extend the development, appendices which explain more useful details, etc. As a teacher, I find it more effective to cooperate with the text rather than to supplement it or to have it supplement me. Our core students largely aim to become problem solvers rather than mathematicians and so can't afford to do all of everything, but I still feel better leaving a course with a good strong reference text rather than having to later replace my course text with a more usable reference.

Everybody likes applications! They are a perennial cadet favorite; as a cadet, my classmates and I were always very pleased when an application problem, known as "Special Problems" in those days, was assigned. This affinity continues for both students and faculty, and is particularly appropriate for a core population that aims to be confident, competent problem solvers. For as long as I have been on the faculty, we have worked hard and committed quite a bit of ingenuity to creating these, from Undergraduate Mathematics Application Projects (UMAP) modules in the mid-80's, to Undergraduate/Interdisciplinary Lively Applications Projects (ULAPs/ILAPs) after our curriculum change in the early 90's, along with a constant stream of new course projects. In all those years of fun and creative projects, I think my favorite was authored by John Wasko when we were teaching together in MA153 on projecting and managing an inventory of rental trucks which also permit one-way rentals between multiple locations, involving modeling the population of trucks at each location over time and setting prices to encourage two-way returns and arranging periodic rebalancing with hired drivers. Another notable cadet favorite was modeling retirement pay and TSP scenarios over a career; students were genuinely enthusiastic about mathematics after that problem, though many wondered why they repeated the same problem as yearlings in Economics. We occasionally posed these at the beginning of a block of instruction and used the problem to motivate the techniques of the block and allowed students to work on the problem during the block and complete and submit it at the end; I always liked this idea and it worked fairly well, but was never the wild success we thought it might be. In the 90's, many of our application problems were developed jointly with our partner departments. These were both great fun and also good for the Academy; like committee work, the products were valuable but the relationships formed were even more so.

The department has a long history of pedagogical use of technology, from the academy's founding with blackboard use in lectures and later in recitations, and later with calculating machines dating from at least the 1930's. (Long-time departmental secretary Frieda Clogston tells of using the department calculating machine to keep score at bowling socials in the 60's.) At about the time that the department was implementing

Derive as our core course computer algebra system in the late 1980s, Jack Robertson used a mixture of research funds and corporate donations to almost single-handedly assemble a departmental Unix (SunOS and AIX) network, with classroom lab. In 1990, he loaded Mathematica 1.1 from a 2 diskette distribution onto our Unix machines, and I was hooked. I appreciate the way that a good CAS allows me to pick my battles, or probably more appropriately, to narrow my focus to those aspects of a problem that I want to concentrate on. I find that I have to be judicious in the classroom; students sometimes worry that we have allowed the software to do something that they need to be more familiar with. Creative thought doesn't end in the classroom; even outside, we have used technology in creative ways to improve the core program. In 1991-92 the Dean's Office scheduled the plebe math course to teach two hours in the morning and two in the afternoon, with a new lesson every day. There was very little duty time between the end of one day's lesson and the beginning of the next to develop the new lesson. A few of us met with the Registrar to discuss this problem; he, as an O6 and a senior operations researcher, explained to us in detail all of the constraints that applied to plebes' schedules and kindly informed us that his was the only feasible solution, but that we were free to search for alternatives if we wished. We came back, thought about it, issued a department-wide challenge with a Hotel Thayer brunch as prize, and within a week had a software-assisted solution for scheduling the plebe course into the four morning hours, where it has stayed since. Just a year later, we decided that we wanted to create a few top sections but also improve collaborative study in the barracks by sectioning the rest of the course by company; when the Registrar regretted that he was unable to generate a sectioning scheme to do this, we imported their PC-formatted data into our workstation (which Jack had named "Euler", of course), wrote the scripts to generate our own ordering, then exported back into the Dean's PC program to produce our own custom section rosters.

Our view of the role of the core math program in the institutional curricular model has sometimes been fundamentally different from that of the other MSE departments. "Client" departments have sometimes viewed (and depicted) us as the foundational slab of the curriculum, supporting the physics and chemistry slabs which in turn support the engineering science and then the engineering slabs. This model emphasizes our (and others') purpose as filling follow-on disciplines' input with our competent output. Not to deny this as part of our mission, but in the mid-80's I appreciated the very different view we were introduced to in our first week as new instructors. We were issued several pieces of required reading in a folder titled "Philosophy of Teaching", one of which was an article from the Winter 1959 ***Assembly*** by Dept Chair Charles Nicholas. His "Grant at Vicksburg" was neither what I expected a Mathematics Professor to write nor what I expected to be asked to read as a teaching philosophy of mathematics during this most impressionable time as a new instructor. Nicholas described the purpose of the core program as developing young minds to be confident problem solvers: to develop mental patterns of supreme confidence to figure out an original solution of an unexpected problem. To be not cluttered by detailed precepts or routine formulas, but instead to possess the ability and experience in thinking fundamentally. To be able to apply first principles of mission and objective, fixation and concentration, maneuver, and pursuit and annihilation. He used Grant's reasoning and actions at Vicksburg to illustrate the purpose of the core math program as developing the mind of future problem solvers, independent of immediate application, and the purpose of mathematics class as being to exercise the mind each day. This was exactly in line with Chris Arney's later characterization of the purpose of the core mathematics program as developing confident, competent problem solvers, not just for USMA, but for the Army.

We understand that there are many challenges involved in teaching in the core program. We have all experienced those stretches when developing a new lesson every day, teaching it three or four hours in a row, frequently grading and giving feedback, and repeating for a seemingly endless period starts to numb us. But we veterans of the core classroom look back at the big picture of these opportunities and experiences as some of our favorite things.

Where Are They Now?

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Introduction

THE whimsical title of this article is a reference to the semi-regular television specials that track the whereabouts of former movie stars that seem to have vanished from the public eye. While some darlings of the silver screen fade into obscurity, other actors have an enduring quality that results in long and varied careers. The celebrity junkyard is littered with stars that have disappeared completely, whose careers are in various stages of demise, or who are experiencing a career resurrection. Much the same can be said for the latest pedagogical trends designed to revolutionize college mathematics education. The mathematical A-list includes such icons as graphing calculators, computer algebra systems, writing to learn, group projects, collaborative learning, clickers, applets, journals, gateway exams, electronic/interactive texts, and many more. Just as at any given time there is always a host of new actors or former stars hoping to become the next big thing, the same can be said of mathematics education.

This brings us to the educational stage of the Mathematical Sciences Department at the United States Air Force Academy (USAFA).   For the past seven years we employed many of the aforementioned stars. Thus the question we wish to answer in this paper is, "Where are they now?" But we also want to point beyond our current status towards where we may be headed.

Previous Performances

During the 2000-2001 academic year we began to think more comprehensively about how we might integrate technology in our courses. While many superb actors were being employed in the national mathematics community (computer algebra systems, collaborative learning, interdisciplinary data modeling, calculus reform ideas, etc.), our local situation looked more like a black and white television set in the age of color cable--four exams and a final with a healthy dose of multiple choice questions.

USAFA had just begun to require all students to purchase laptops and we thought that this provided an educational opportunity that should be taken advantage of. We realized that this opportunity was not simply a chance to integrate technology into our classrooms, but also an important opportunity to bring changes to our pedagogy, assessment, and curriculum. Thus we wanted to bring to bear locally what others were experimenting with in the larger mathematics community.

We began to operate with a philosophical refrain of, "technology is a tool that should be available to students almost always," and our daily pedagogy started to reflect this. Students were encouraged to utilize applets, Excel spreadsheets, and computer algebra systems during class and on exams. Student ability to visualize mathematical constructs with technology quickly increased. And we were able to design exploratory and discovery activities that took advantage of technology's ability to make rapid calculations and/or graphs during our daily classroom routine.

Our assessment of student learning also changed. Since technology could easily calculate answers to skill-based questions, we had to approach exam writing knowing that questions along the lines of "compute ..." or "calculate ..." might be rendered meaningless in the presence of powerful software. This meant in both our curriculum and assessment, we placed more emphasis on conceptual understanding over the ability to rapidly reproduce basic skills. We realized that interdisciplinary projects were natural opportunities not only to leverage technology, but to assess critical thinking, modeling, and problem solving skills. While we did not initially plan to incorporate writing as a component of our assessment portfolio, we began to see that asking students to write about their thought processes while interacting with technology was a very helpful activity. This realization eventually morphed into a widespread use of writing as an assessment instrument to the point that our first two calculus courses were designated "Writing Intensive" by our institution.

Lastly, our course content also changed to reflect the use of technology as well as our new emphases. We began to utilize discrete dynamical systems as a modeling tool. We adopted a new textbook to reflect our emphasis on conceptual understanding, modeling, exploration and the "rule of three." And, we brought some multivariate calculus ideas forward into differential and integral calculus where there seemed to be meaningful ways to extend student thinking as much as possible.

Eventually, five service courses (Differential Calculus, Integral Calculus, Multivariable Calculus, Differential Equations, and Engineering Mathematics) were changed to incorporate these new stars in pedagogy, assessment, and our curriculum. Although some of these stars were closely tied to our desire to meaningfully use technology, many of our actors were not. But all of these changes reflected trends in the national mathematics community at the time.

The Current Act

These new actors were, in most cases, substantial departures from those in our employment at the time. As is usually the case with large change, not all of what we did worked out as we originally intended. Students formed a faithful repository of strong resistance and some of our faculty were uncomfortable with many of the changes we wrought upon them. When we asked students to deal with economics in an interdisciplinary project they complained that, "this isn't an economics course." When we required them to explain their ideas using writing, they leveled a similar complaint: "This isn't an English course." Getting the various forms of technology we were using to work properly was, at times, a headache for all involved. Instructors in our department were not sure they could properly grade student writing. Our increased emphasis on conceptual understanding raised eyebrows among faculty within and without our department. And utilizing discovery and exploratory techniques as a meaningful part of the lesson plan was awkward for some who had not learned mathematics in that way themselves.

Eventually other people came along behind us to modify and improve upon our initial efforts. We began to utilizing online software to administer gateway exams in order to address concerns brought about by our increased emphasis on conceptual understanding. A great deal of administrative machinery was put in place surrounding group projects to ensure that the intent of collaborative work was being maintained (project days, project quizzes, group self-evaluations, etc.). We joined other departments in a Writing Across the Curriculum effort and attempted to codify how we should do writing in our five service courses so that students and faculty would focus on learning mathematics. Holistic grading rubrics were developed to address concerns of consistency and efficiency in grading writing.

Initially, these changes did little to quell student discontent with our pedagogical celebrities. However, concerned faculty began to accept these changes to the point where discussion moved away from, "Why are we doing this?" to the more preferred questions of, "How can we do this better?" and, "How can we sustain what we’ve started?"

As others improved upon our changes, they naturally had their own ideas and these new ideas made their way into courses. But, little was removed when new ideas were added. In short order, our courses became packed full of many interesting ideas but with little focus. This resulted in administratively complicated courses with many moving parts. Newer faculty had little understanding of some of the rationale that drove initial changes and unintended negative consequences of some of the newer ideas started to occur. Given the high turnover in faculty at USAFA, our courses became very difficult to sustain.

At this point, student complaints about the utilization of technology, writing, interdisciplinary projects, etc. were few. However student frustration with the complex administration of the course began to negatively impact their willingness to grapple with the complexities of the mathematics. They were confused by multiple requirements happening at the same time. Students were expected to upload homework via their tablet pc to a course website while doing other parts of the homework using online software, while preparing to write about yet another homework question to be turned in on a paper copy, while a multi-page project loomed on the horizon shortly after a major exam.

The New Characters on the Horizon

Because we are committed to improving how we help our students learn mathematics, we are now in the process of re-evaluating the actors that we have relied upon in our pedagogical play over the last several years. Our department remains firmly committed to the use of technology "almost always" in our courses. We see writing as an important enough tool that it has become part of our departmental learning outcomes and have moved to a more developmental philosophy with shorter, more frequent assignments. We value collaborative learning and interdisciplinary projects but are asking the questions, "Are we getting what we want out of our group projects? Can we utilize these in a more effective way?" We see a continued role of importance for gateway exams but are re-visiting the content and logistics. Each of these players are being viewed through the lens of, "Are we able to implement each of these in a sustainable way?"

Technological nirvana has yet to materialize for us. We do not have seamless interaction with technology in our courses. We don't know how to write good exams that *require* technology nor have we given enough thought about the implications for changes in the course content that a widespread implementation of technology might infer. We want to move beyond thinking of technological instruments as powerful calculators and more towards utilizing them as vehicles of inquiry, discovery and modeling. We see tremendous untapped potential for technology to drive higher order learning in our courses.

Future possibilities for change in our pedagogy, assessment, and curriculum include the possibility of deemphasizing the course content in favor of more emphasis on modeling and problem solving. So instead of teaching students differential calculus and integral calculus, we would teach students how to frame, model, and solve problems using not only differential and integral calculus, but other types of mathematics as well. This change has significant implications within and without our department. But we believe that it deserves serious consideration for the future before being summarily dismissed.

We have been and are currently trying to improve how we help students learn mathematics. This is necessarily an iterative, non-linear, ill-defined, constrained, and pained process. We believe new actors will come onto our learning stage but we also expect some of our old friends to re-invent themselves and make grand reappearances. We are committed to sustainable change in the hopes that our students will learn more and grow close to the ideals that we hold forth for our cadets.

Ranting for CAS: Longing for the World to be Fully in Post Computer Algebra System Emergence

Dr. Brian Winkel
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I HAVE been truly blessed to teach at West Point for 14 years and this year I am a visiting professor, make that Distinguished Visiting Professor (tut! tut!) of Mathematical Sciences at USAFA here in gorgeous Colorado Springs. I now get to teach with yet more wonderful and committed young people in class and with young (at my age, just about everyone I engage with is young!) faculty as a mentor and colleague. I am learning many new things, great assessment strategies which will emerge in pieces in this forum I am sure, and how teams work out here for success of cadets and faculty.

All the while I was at West Point I had no windows in my office. Indeed, there are only a very few windows in all of Thayer Hall, a main classroom and office building at West Point, where my office and classrooms reside. Below is the view from my office at USAFA. Awesome!!! I get to see the terrazzo where all the action is as cadets run (the first year students are required to do so, and on specific narrow marble paths) or leisurely stroll – the life of the "upper class" – to and from the dorms (they are not called barracks

the way they are back at West Point) to the main classroom building (Fairchild Hall) where my office and classrooms reside. I guess this view is what the word "Distinguished" translates to.

So this rant needs to get going. Let's get a positive first derivative on volume and vitriol, and get it on!!! Perhaps this gorgeous sunlit view I sit before as I write has helped shed some light on what I have been experiencing these past 20 years with respect to the institutional and collegial use (mostly lack thereof) of technology in undergraduate mathematics teaching.

I was blessed to be able to use the computer algebra systems (CAS) *Maple* on VAX workstations back in the 80's and *Mathematica* on NeXT workstations in an innovative curriculum I helped design and teach. I found working with CAS liberating. I changed much in what I did and how I saw things for teaching. As an example, in the past (I will refer to this emergence as PreCASE – Pre Computer Algebra System Emergence) when trying to convince (and that is what we really do on first pass mathematics in introductory calculus courses) students that the derivative of $f(x) = \sin(x)$ is $f'(x) = \cos(x)$. I had to deal with the sine of the sum of two functions in considering sin(x+h) in the difference quotient

$$\frac{\sin(x+h)-\sin(x)}{h}$$

before considering the limit. Then I had to do some type of pinching theorem with very nice geometry to be sure just the correct terms disappeared as we take the limit. Have you been there? Are you still there? It ain't all that bad, it is kind of elegant, and it gives students a sense of the beauty that drew us to mathematics.

Now, if we stop every time in every topic to tell students how mathematicians think and appreciate the world around them then for those 99% of the students who will NOT be like us (or Be Like Mike – referring to Michael Jordan of NBA fame) I believe we are giving them news about something they do not care about, something that obstructs learning for them, and something that could be replaced with empowering ideas and approaches to the subject they CAN relate to and wrap their hands and minds around.

In my own PostCASE – Post Computer Algebra System Emergence – I simply discuss the geometry (students always respond to pictures) and we plot three graphs on one set of axes:

1) sin(x),
2) (sin(x+.00001)-sin(x))/.00001, and
3) cos(x),

with (2) being the PGA (pretty Good Approximation – they agree – of the derivative of sin(x)). Guess what happens? They say there are only two graphs. We replay Sesame Street – "One of these things is not like the other one" – with a new twist "Two of things are the same thing." Bingo! They are convinced (and that is what we really ought to be about – convincing, not necessarily proving - in the first, and in some cases, last round of calculus) that the derivative of sin(x) is cos(x). This PostCASE viewpoint is truly different and, in my humble – well not really so humble – opinion, truly better. It certainly is very convincing, it is efficient, and it is a precursor to how we shall use the CAS throughout the course I am preparing for them to discover, learn, and do mathematics. The PostCASE approach empowers instructors and cadets alike and when things go wrong – and they most assuredly do – due to syntax, glitches, numerical issues, etc., then this presents the most opportune time for true inquiry into what is really going on, perhaps deeper than you would otherwise go. Take advantage of minuses and make them plusses.

When folks talk about technology in the classroom, they get bent out of shape about using overhead projection of computer images, PowerPoint, SynchronEyes classroom management software to check out and put up student efforts on the front board from resident laptops in the classroom, smart boards so what the instructor does is captured by the technology, so students can run through it when they are awake outside of class, etc. Don't get me wrong, there are some real advantages to using these new approaches and I have watched some of the young Captains and Majors at West Point really "work the room" as well as Sinatra worked Vegas showrooms. Terrific!!! However, when I talk about technology in the undergraduate mathematics classroom I mean DISCOVERING and DOING and COMMUNICATING mathematics, not watching it.

So what do I do? In and out of class, with *Mathematica*, I play the sound of a solution to a differential equation and experience the feel of a damped oscillation while also listening to beats and resonance. Students see the analytic or closed-form solutions and relate it to the by-hand work we did in simple examples to gain confidence that this mathematics does work in a real world. We routinely animate mathematical objects with changing parameters – not variables – so we can see the effect of the numbers in a function or model.

Most importantly, cadets do their mathematics inside *Mathematica* and write-up their results in the same CAS, communicating in a professional manner with all the connectives in thought and process displayed, even to the extent of using correct fonts and symbols. One future civil engineer in my Engineering Mathematics course was somewhat "overboard" about getting things just right. She said that when she is a PE (Professional Engineer) and signs her name on the bottom line someone has to take over the implementation from what she communicates and it just has to be good. She liked making it good, clear, neat, and elegant as well as correct!

Let me give you but a few small examples of how CAS can influence or revolutionize (depending upon how far you let it carry you) your teaching, perhaps as it has mine or perhaps in different directions. Illustrations of what I have in mind can be realized in articles I have published over the years. Pointers to, indeed copies of, these pieces can be found at the following web page:
http://www.dean.usma.edu/departments/math/people/Winkel/ .

In an area of the calculus course I call “Elements of Stuff” I present many scenarios which lead to integrals as a good problem-solving strategy. I shuffle up applications – staying away from cook book calculus texts whose section titles alone put students into algorithmic slots rather than develop their general strategy approaches. In this way students are not guided by method of this (shells) or that (washers), but rather they see the process of cutting something into little pieces, identifying the nature and the computational form of those pieces, and then building an integral to address the issues of concern. Cadets just slash away at situation after situation where each one calls for an accumulation to build an integral out of little elements of stuff. Having built the integral they then use the power of CAS to evaluate it or to study changes in parameters and finally explain how they built the integral (in *Mathematica*, explaining what they did in their own words interspersed between their analyses) and what their results mean in the context of the situation under study. This is doing mathematics with the right perspective, not concentrating on by-hand symbol manipulation, however, “good for the mind” or “professor favorite” we have in mind might have been PreCASE. By-hand work is in the eye and mind of the beholder, e.g., the downstream physics instructor who storms your office and gripes because the students cannot take the derivative of $e^{-3t}\sin(3t)$ by hand and does not see the ability of this student to do, organize, and communicate with a tool like *Mathematica* which can easily do this derivative. Moreover, what by-hand work is essential? Does the complainer want the students to continue to use cuneiform tablet writing techniques, use log tables, extract square roots by hand, solve cubic equations by hand, grind out long tedious solutions to second order ODE’s with drivers? When will the complainer let go and move ahead?

In modeling, engineering, and calculus classes I have had students discover Fourier series by examining the trigonometric series approximations (we use the imagery of sound generators trying to reproduce a signal) for simple functions like f(x) = x in the interval [-π, π]. They set the criteria for best fitting, compute coefficients, and look for patterns. We do this using inductive thinking, looking for patterns BEFORE we derive (prove!) the more general Fourier coefficients and then we use this all to build, play, and analyze sounds in *Mathematica*.

When other classes are working their way through the shift theorems of Laplace Transforms we are using *Mathematica*’s **`LaplaceTransform`** and **`InverseLaplaceTransform`** commands to translate the problem from calculus into algebra (frequency domain as the engineers refer to it) and back with solutions to interesting modeling questions in studying system responses to various inputs and parameters in those inputs. I had a very bright senior once ask me for help. (At West Point we call it AI while at USAFA we call it EI and what do you folks out there call it at your Academy?) He was working through all the integrals and properties of Laplace Transforms in preparation for the overall exam on the subject. When I asked him what the big picture was he had no clue – and he was a sharpie. He was lost in details, details that probably should not be given out in such doses at this stage in his career, certainly not so in light of the CAS technology which could do so much for him and pull him through the dark cloud of myopic calculations and ground out (if only they could field and throw AND HIT as well as Derek Jeter) manipulations. He did not even understand or have a sense as to one of the reasons we study these transforms as an example of the beautiful notion of transformation from one domain (time - t) to (frequency - ω). It does not have to be that way anymore if we but embrace technology and move into the PostCASE.

Years ago I shepherded a National Science Foundation grant, Complex, Technology Based Problems in Calculus (see http://www.rose-hulman.edu/Class/CalculusProbs/ for many rich illustrations), in which we created problems which made effective use of CAS technology. From time to time I revisit them with my students and report on this sojourn in the literature with published papers. One such example I wrote up [2] involved the following question. When you submerge a ball in water, how much more of the ball can you see from a given point above the surface of the water because of the way light "bends" or is refracted due to the water? Students are intrigued by such a problem and the CAS tool assists them, but only after they struggle with problem formulation, with a modicum of analysis, and with lots of geometry and optimization issues. In another example [1] I used to motivate – the students discovered it because of a need in the problem – the gradient to a surface by asking them to fully describe exactly what they could see on one mountain surface from a point on the face of the opposite valley slope. GPS has provided many questions like this, such as what is the minimum angle from the horizon where you can see space from a spot on the Earth. With CAS these problems become reachable, serve as motivators, and convince students that mathematics is useful and within their grasp.

CAS is NOT a push button approach, on the contrary, my friend, it is often an end game confirmation after much analysis or it can serve as a front end to discovery and general notions, but most likely – if done right – it is an intimate part of the problem-solving kit we need to provide for our student. We do not need to just provide it; we need to work it, to practice it, to see its implications for doing mathematics, and for teaching mathematics.

Frankly, I think we are in some modest retreat from fully realizing the potential of CAS and I would urge everyone to "Get in the Pool!" Even at the two Academies I have taught at I see less than adequate attention paid to CAS in faculty development efforts, in course planning, even in project offerings, and certainly in exams. This would be excusable if it were not for the fact that it is an indication we are looking backwards, not forwards, and the fact that we are not really using our own intellect as teachers to see how to effectively incorporate this tool. At times, because of tight lock-step teaching and testing we have in place we are actually crushing the creative spirit of those who would embrace PostCASE. I have felt it and it is chilling. It ought not to be. We should embrace this very engaging technology and share it much more with our students. I believe we do them a disservice by not taking them on the journeys CAS will permit and bringing their tool kit for mathematics up to date, beyond pen and quill, beyond slide rule, beyond calculator, and beyond by-hand effort, by taking them into the light of "What have we here?" Or "I wonder if this." Take your students on more "What if?" journeys with CAS today and then do it tomorrow as well. They will enjoy it, they will be better for it and so will we.

Listen, I appreciate the beauty and abstraction as much as the next Ph.D. mathematics geek, after all I studied Noetherian (noncommutative) ring theory, so I know a thing about theory, about beauty, about elegance, about precision in thought, etc. However, I think we owe it to our students (almost none – dare I say - or we could say a.e. for almost everywhere which you might remember from your measure theory studies – of whom will be like us) to show them the amazing and great things mathematics can do, not the view down the deep well of nuance, counterexample, theory, and tight well-crafted arguments, a.k.a. proofs. In PostCASE, if we but use the technology such as *Mathematica* or *Maple* we can see the structures we love so much AND permit intellectual and skill growth in our students. Will the skill set still be the same, i.e., will they extract square roots by hand (I did in my youth – but then we had the slide rule and eventually calculator) and will we make them do trigonometric substitutions on integrals or will we open up a huge wonderful tool box like a quality CAS and let their creative juices flow as they discover, do, and communicate mathematics and its application? I think we need to leave the PreCASE times and move ahead to PostCASE, with some look backs longingly at things we loved and practiced. I have found that in looking ahead with CAS I have new horizons and prospects open to me and my students and I believe we all would if we could just take the plunge and enjoy what CAS has to offer us and our students, perhaps sometimes recklessly, but with some prudence in our dive.

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Operations Research at the Naval Academy

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UNTIL the late 1970's the Naval Academy issued degrees in Operations Research (OR). The reasons for the major being dissolved are not exactly known, but there are theories based on some CNA results about the Nuclear Navy. Since then there have been several OR courses taught (Linear Programming, Logistics, Stochastic Modeling, Simulation, Probability Theory, and Statistics) to support the applied mathematics and the quantitative economics majors.

In 2002 the Navy began a subspecialty coding system for officers based on their undergraduate education. The first code established is the 3211E code to support the Navy's need for officer analysts. The code identifies junior officers who have analytical skills and training that qualify them to fill designated shore billets that involve analytical studies important to the future of the U.S. Navy. Officers with a 3211E code will be given special consideration for assignment to Naval Postgraduate School (NPS) in Monterey to earn a master's degree. Dr. Charles Mylander, Professor Emeritus at the Naval Academy, was one of the designers of this program. The E-code has both course and grade point average requirements. Currently the majority of students earning the E-code are majoring in either Applied Mathematics or Quantitative Research.

In 2007 the first OR Capstone course was taught by LCDR Kyle Kliewer, USN (Ret). This initial Capstone course was in War Gaming Theory and was a huge success with the midshipmen and with the military OR community. Subsequently, in 2008 and 2009, OR Capstone courses were offered in Integer Programming and Game Theory. The third OR Capstone is planned for the spring of 2010 in Network Theory. Additionally, several Mathematics Honor Students, Bowman Scholars and Trident Scholars have completed research in the OR field.

Many midshipmen have completed graduate education in the field of OR over the past several years. Currently two graduates of the Naval Academy class of 2009 are beginning master's degree programs in OR: one at Massachusetts Institute of Technology and one at the Colorado School of Mines.

Outside the classroom midshipmen are participating in OR academic endeavors. Some are increasing their OR experience through internships as part of their summer training. This past year two midshipmen interned: one at Johns Hopkins University Applied Physics Laboratory in the Underwater Division and another at the Naval Surface Warfare Center. Others are participating in educational competitions, such as the one hosted by the Military Operations Research Society. All of these students gain insight into the group dynamics and the scope of problems being addressed in the world of Military OR.

This resurgence in OR has led to the development of a proposal for a degree in Operations Research at the Naval Academy. This degree, if approved, will provide greater opportunities to show the value of OR to the midshipmen. While the proposed major does include the traditional courses mentioned earlier, it also contains advanced courses in optimization and statistical modeling. Students who pursue a degree in Operations Research would take all the required courses for the E-Code and have a stronger background for summer internships supporting the Navy's analytical objectives. The degree will provide the strong mathematics background necessary for graduate studies, but will be based on applications and solving real-world problems. Since the proposal was initially put forward, courses in Spreadsheet Modeling, Combinatorial Optimization, Nonparametric Statistics, Stochastic Epidemic Models, and Analysis of Experimental Data have been taught.

So while currently there is no Operations Research Major at the Naval Academy, the discipline nevertheless remains very much alive and has a bright future.

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