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A Note from the Editor

This issue features some diverse and interesting approaches to *Assessing and Remediating Fundamental Skills within the Core Math Program*. If you find something noteworthy, I encourage you to direct questions and comments on these topics to the authors and/or consider submitting a follow-up or response for the Spring 2012 issue.

Calling all Academies!! We value and welcome insights from all of our service academies' mathematics departments for publication *Mathematica Militaris* in order to grow from shared perspectives.

The upcoming Spring 2012 issue will emphasize to the topic-- *Beyond the Core Math Program: Math Electives that Service Engineering Disciplines*. At your service academy, how many more math courses must an engineering major take, compared to a humanities or social science major? Do majors in different engineering disciplines take different service courses? How is coordination done to support the discipline's needs? How does the ABET accreditation process affect these courses?

Submit an article! As always, we also continue to welcome general topic papers you wish to submit. The Editorial Policy is included at the back of this bulletin. March 10, 2012 is the deadline for submissions for the Spring issue. We look forward to hearing from you!

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Assessing and Improving Students' Fundamental Mathematical Skills

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Introduction

In 1834, the United States Military Academy Committee on Military Affairs concluded that "[mathematics] is necessary, both to impart to the mind that combined strength and versatility, that peculiar vigor and rapidity of comparison necessary for military action, and to pave the way for progress in the higher military sciences." However, the committee cautioned that "[the mind] should be taught gradually to develop its own powers, and as it slowly learns their capacity and the manner of employing them, the increasing lights which are thrown upon its course will enable it to go on for an unlimited extent in the path of improvement."[1] While technology has changed since 1834, fundamental mathematical concepts continue to serve as a foundation for students throughout their Science, Technology, Engineering and Mathematics (STEM) education.

Our method for assessing and improving students' fundamental skills begins when students receive a letter of acceptance to the United States Military Academy (USMA) and continues into the STEM disciplines. Approximately half of the cadets at West Point will be STEM majors, but every cadet will have a four-course mathematics sequence and a three-course engineering sequence in order to fulfill the requirements of a Bachelor of Science degree. Considering students enter West Point with different backgrounds in mathematics, it is important to establish a common baseline of mathematical knowledge from which to build the curriculum.

The Department of Mathematical Sciences has identified 13 fundamental concepts in which cadets must demonstrate proficiency as this baseline of mathematical knowledge. Identifying those concepts begins with an assessment of West Point's ten intellectual domain goals. These are the overarching goals of the academy to support producing graduates who anticipate and respond effectively to the uncertainties of a changing technological, social, political, and economic world. Since the ten goals are not isolated to any specific course, each intellectual domain goal has a goal team to ensure that the goals are embedded within the curriculum across the core courses and majors.[2] The Engineering and Technology Goal Team and the Math and Science Goal Team, assisted the Department of Mathematical Sciences to identify the 13 concept areas.

Students are able to demonstrate proficiency in the 13 concepts areas through a series of Fundamental Concept Exams (FCEs). In the subsequent sections, we will discuss the sequence of exams and the standards for validating the exam. Then, we will explain the numerous resources available to students. Finally, we will discuss ways to track which questions should appear on the exams and ways to track student usage of the website and other resources.

Sequence & Standards

Our experience indicates that if students lack a good understanding of 13 fundamental pre-calculus skills and concepts that we have identified, they will most likely not perform to their full potential in subsequent math, science, and engineering courses. The 13 fundamental concept areas are:

1. Algebra and Real Numbers
2. Radicals and Exponents
3. Algebraic Expressions
4. Factoring and Prime Numbers
5. Systems of Linear Equations, Inequalities and Absolute Values
6. Polynomials and Rational Inequalities
7. Equations of Lines
8. Functions
9. Quadratic Equations and Systems
10. Trigonometric Functions
11. Logarithmic and Exponential Functions
12. Graphs and Graphing
13. Analytic Geometry

Each of these areas is further divided into specific objectives. For example, under the Logarithmic and Exponential Functions concept is the objective "Solve simple logarithmic and exponential equations (e.g., solve the equation $3^{x+4} = 4$ for x)." The added specificity allows the students to focus their studies.

Shortly following his/her notification of acceptance to the United States Military Academy, a cadet candidate's journey through the core mathematics sequence begins. The core mathematics sequence is a four-course sequence consisting of mathematical modeling and introduction to calculus, differential calculus, integral calculus, and probability and statistics. A cadet candidate's first contact regarding the Fundamental Concepts Exam (FCE) requirements is through information mailed from the West Point Admissions Office. They are led to the fundamental concepts website[3], which informs them of scheduled evaluations in fundamental mathematics skills following their arrival and the standards required to achieve a passing mark. We will discuss this in more detail in the Resources section of this article.

Contacting cadet candidates prior to arrival at USMA has proven successful for both the Department of Mathematical Sciences and the cadet candidates. As a critical building block to math, science, and engineering courses, knowledge of these basic mathematical skills allow USMA instructors to focus on more advanced materials. Most incoming cadets are undoubtedly eager to face the challenges West Point offers, and this provides them an actionable outlet to do so prior to their arrival. Additionally, most cadet candidates recognize their period at West Point will be rich with experience, but short on free time. Permitting them the opportunity to prepare for forthcoming academic requirements allows a cadet candidate to alleviate some of the stress from a very busy first summer at the academy, while developing a more thorough foundation for their collegiate mathematics careers.

Clear expectations of testable fundamental concept skills and standards to achieve are conveyed to cadet candidates prior to their arrival at West Point. An 80% or better is required of all students on an FCE in order to validate. This standard remains consistent throughout a series of opportunities available throughout their Plebe year. A pencil and scratch paper are the only authorized resources for the exams. Algebra, analytic geometry, and trigonometry skills tested often require formulas committed to memory. Memorization of this material is included among the basic building blocks to the foundation of mathematical knowledge a cadet will need to successfully complete his/her core mathematics classes.

A series of evaluations begins shortly after cadet candidates arrive at USMA to begin their summer of Cadet Basic Training. The Summer Gateway Exam, given in early July, is the first FCE and therefore taken by all cadets. The exam consists of 30 multiple choice questions and 5 questions with work shown – a suitable number to gauge each student's proficiency in the fundamental concepts.

Results of the Summer Gateway Exam are taken into consideration before the academic year begins to assist sectioning cadets by ability level. Those who struggled significantly on the exam may be enrolled in MA100 (Pre-calculus Mathematics) where they will continue to develop their fundamental skills prior to entering their USMA core math sequence. Conversely, demonstrated proficiency on the Summer Gateway Exam – while not the only indicator – is a required prerequisite for selection into the advanced track of core mathematics beginning with MA153 (Advanced Multivariable Calculus). All cadets in the advanced mathematics sequence validate differential calculus and therefore take a three-course sequence.

Fundamental Concept Exams continue throughout MA103 for those who do not achieve an 80% on the Summer Gateway Exam. A total of three subsequent FCEs are offered to that population, roughly once per month throughout their first semester. These exams are each 20 questions with all work shown (no multiple choice). Although the Summer Gateway Exam does not contribute to a cadet's academic grade point average, subsequent FCEs do impact their MA103 course average. They account for 100 of the 2000 total available MA103 course points (5%), with only the highest of all FCE grades achieved recorded per student. Although cadets scoring at least an 80% on the Summer Gateway Exam are not required to take the FCE, if enrolled in MA103 they have the option to take it for score to improve their course grade. Additionally, any cadet who already achieved a passing mark of 80% or higher can continue to take the exams in an effort to improve his/her grade.

As a universal requirement, any student failing to achieve an 80% or better throughout all four opportunities (Summer Gateway Exam, and three subsequent FCEs) is at risk of failing MA103. These students are addressed on a case-by-case basis. If allowed to proceed with their core math sequence, a deliberate remediation plan is agreed upon as terms for their continuation. Students failing MA103 fall semester will join those selected to enroll in MA100/101 to take MA103 in the spring. Completing the core mathematics sequence for that population will likely require a summer semester enrollment in order to catch up to their peers.

Resources

Students are provided numerous resources to help them prepare for the fundamental concepts exam. Shortly after they are offered admission to West Point, high school students are provided a link to the fundamental concepts website. This website gives detailed descriptions of all 13 concept areas, consolidates videos and reviews of these concept areas, and links nine sample Fundamental Concept Exams. Cadets regularly comment on the usefulness and statistics appear to support their claim. In just over seven months, this website saw over 6,300 visitors.

In addition to the fundamental concepts website, students who fail to validate the Summer Gateway Exam have online practice problems and practice exams for the first 12 lessons of the semester. These practice problems are administered via a commercial online assignment that provides real time feedback to the students. If students do not understand how to work a problem, they can access links that allow them to read sections of a textbook that explain the concepts, watch similar problems being worked, and even chat with a tutor 24 hours a day.

During the first three weeks of class, before the first fundamental concepts exam of the semester, some instructors will spend class time reviewing an "FCE minute" problem of the day, or short practice problem focused on a required FCE skill. In addition to this classroom instruction, students have the ability to schedule additional instruction with their instructors. Each cadet company has an established company tutoring program where upperclass cadets can volunteer to tutor for particular subjects. Cadets identifying themselves as math tutors regularly tutor Plebes on fundamental concept areas. While the responsibility for demonstrating proficiency on the fundamental concepts lies on the individual student, they are provided numerous resources to that can help them with this requirement.

Tracking

Course-wide Fundamental Concept Exam statistics are utilized before and after each exam. Prior to the exam these statistics assist in designing questions. Each of the 13 fundamental concept areas has multiple objectives, making it infeasible to test all on every exam. Therefore it is important to deliberately compile exams with a representative cross-section of questions addressing the required breadth of knowledge, and ensuring diversity among historical archives of exams. While a diverse collection of questions is important, testing different concepts on each exam is not necessarily the objective. Fundamental skills have varying levels of importance, as well as levels of difficulty, and all these factors are taken into account when preparing an exam. Certain skills appear on the majority of exams. For example, as seen in Table 1, Skill 2b – to manipulate algebraic expressions that contain integer and rational exponents – was tested on every FCE iteration in the Fall 2010 and Fall 2011 semesters because it continues to be a weakness of students and is a key fundamental concept.

Table 1. FCE tracker for the first semester of academic years 2011 and 2012.

Concept	AY11-1				AY12-01	
Area	July '10	Sept '10	WPR 2	WPR 3	Aug '11	Average
1a	94.62%					94.62%
1b	77.02%	78.54%			87.08%	80.88%
1c						NA
1d		87.44%			94.86%	91.15%
2a	61.16%		67.13%			70.43%
2b	70.07%	73.79%			78.16%	79.40%
3a	83.56%	96.60%			83.59%	87.92%
3b	87.64%	91.47%			93.04%	90.72%
4a		84.50%			92.59%	88.55%
4b	71.27%					71.27%
5a	89.60%				92.46%	91.03%
5b	72.58%	86.05%			89.10%	82.60%
5c	66.84%	77.49%				72.17%
6a						NA
6b		72.44%				72.44%
7a		91.38%			83.45%	87.41%
7b	78.58%	92.70%			87.45%	86.24%
8a						NA
8b	68.00%		48.43%		62.38%	62.53%
8c	90.11%	89.72%				91.33%
8d	93.67%				85.20%	89.44%
9a	65.60%	81.85%			75.15%	74.20%
9b		80.53%				80.53%
10a	84.00%				95.13%	89.57%
10b						NA
10c	47.27%	80.50%		63.59%		63.79%
10d	82.56%					86.03%
10e						NA
10f						NA
11a	76.51%	82.50%			78.52%	79.18%
11b						NA
11c	86.62%	95.26%				90.94%
12a		76.82%			82.33%	79.57%
12b	72.44%					72.44%
12c	63.20%			78.69%		74.13%
12d						NA
13a	76.95%				84.81%	80.88%
13b	88.65%	87.75%			69.77%	82.06%
13c						NA
13d	57.38%	87.59%			67.47%	70.81%
13e	87.93%					87.93%

Following an exam, a report is produced to compare how students performed in each categorization within the 13 required fundamental skills. Recording scores by question on non-multiple choice questions can be time consuming. However, this allows us to identify where positive and negative trends exists in understanding these concepts. Any identifiable trends are shared with the 20+ instructors of the course, and addressed when appropriate. Remediation can be addressed at the course level by incorporating a fundamental concept that scored poorly on the FCE into a subsequent course exam (labeled “WPR 2” and “WPR 3” in Table 1) or course wide project, or alternatively left to instructors to address independently as they deem appropriate. Remediation results using Written Partial Review (WPR) exams are seen in Table 1 for the Fall 2011 semester.

While analyzing student performance is beneficial for crafting the exams and remediating weak areas, analyzing student involvement in the learning experience is equally as important. With such a large number of resources available to the students, we are constantly trying to determine whether or not students are using them and whether or not they are helping students master the fundamental concepts. In order to answer the question of whether or not students are using the resources, we use a web analytics program to track website usage. There are many free, easy-to-use analytics programs available online. With very little experience in website programming, it is very easy to embed these trackers into the html coding of any website. The functionality of these trackers can range from complex to extremely simple. For the purposes of our analysis, we were looking to obtain data on how many people were visiting the website, when those individuals were visiting the website, and where the visitors were coming from.

We tracked the fundamental concepts website usage from March through September 2011 as shown in Figure 1. Overall, there were 6,342 visitors over the seven-month period. More interesting than the total number of visitors was the pattern of visitors during three particular periods, as labeled on Figure 1.

Fig. 1. Graph of website visitors from March-September 2011

The first timeframe of note (indicated by the '1' in Fig. 1) occurred in late-April when we saw nearly 1,000 hits in a one-week period. This occurred after incoming students received the correspondence directing them to the fundamental concepts website. This suggests that the majority of incoming students at least visited the website to view the required fundamental skills for new cadets.

Period 2 shows a gradual buildup of visitors followed by a sudden drop off. The gradual buildup represents the weeks leading up to Cadet Basic Training and the sudden drop off coincides with the start of the summer training when new cadets do not have access to computers. While this is a seemingly obvious result, it allowed us to confirm that the majority of the website users were incoming cadets. This website is open to the public and does not require a password so the visitors do not necessarily have to have any affiliation with West Point. In fact, between March and June, the fundamental concepts website saw users from 19 different countries. This was not surprising; the class of 2015 includes international cadets from 14 different countries.[4]

The final spike (i.e., Period 3) in users occurs around the final week of August and the first week of September when the website saw nearly 600 visitors per week. This coincides with the first fundamental concepts exam of the semester which occurred on

September 7, 2011. This was encouraging because it showed that not only were students using the resources on the website before coming to the Academy, they continued to access the website when preparing for the first fundamental concepts exam.

Over the past three years, instructors have decreased the amount of time they have spent covering fundamental concepts both in and out of class. However, during this same time period, we have seen an increase in the number of students showing proficiency in the fundamental concept areas. From 2010 to 2011, we saw a 31.9% decrease in the number of cadets failing to pass the first fundamental concepts exam. The level of usage as shown in Figure 1, combined with the recent success in decreasing the number of deficient cadets, suggests that students are taking responsibility for their own learning and utilizing the resources they are provided.

Conclusion

According to *Educating Future Army Officers for a Changing World*, the foundational document from the West Point Office of the Dean, the purpose of the academic program is to "establish the intellectual foundation for service as a commissioned officer."[5] Establishing a solid foundation of fundamental concept skills is critical to follow-on success throughout both the core math courses, as well as future studies in science, technology, engineering and mathematics. With cadet candidates arriving from diverse backgrounds and significantly different high school mathematics experiences, the FCE is a necessary calibration tool to ensure the common language of mathematics at a basic level is well understood across the board. Tracking itemized performance by concept area creates a historic record allowing for future analysis, which together with online analysis tools allows us to evaluate student understanding on a macro scale as well as gauge the effectiveness of our approach.

There is undoubtedly room for improvement. The third spike in Figure 1 suggests that many cadets are waiting until the week of the Fundamental Concepts Exam to study the 13 concept areas. To prevent this procrastination, instructors could stress the usefulness of the website from the very beginning of the course. The incoming cadets do not have access to the internet during Cadet Basic Training, but a curve with fewer spikes once the semester begins in August would indicate that students were devoting daily or weekly time to fundamental mathematical concepts rather than simply using it as a study aid for exams. Retention is another area for improvement. Because students can pass a test does not necessarily mean they will retain that knowledge for the next four years. We share information with follow-on courses to ensure they stress areas of weakness in addition to building upon the fundamentals tested in the first course of the core mathematics sequence.

Fundamental Concepts Exams continue to build throughout the core mathematics sequence by adding calculus concepts and eventually probability and statistics concepts. Tracking historical statistics on the FCE provides important feedback allowing the exam to evolve and meet course objectives. As the starting point to USMA mathematics curriculum, the FCE is forecasted to continue as a key building block for cadet candidates upon arrival for years to come.

Endnotes

[1] Extract from report on Military Academy by Committee on Academic Affairs (May 17, 1834), included in the Annual report of the Superintendent of the United States Military Academy, 1896. Washington: Government Printing Office, 1896.

[2] *Educating Future Army Officers for a Changing World: Operational Concept for the Intellectual Domain of the Cadet Leader Development System.* Office of the Dean, United States Military Academy, 2007. http://www.usma.edu/opa/clds/Educating%20 Future%20Army %20Officers_Ver%203.pdf

[3] Prospective Candidate Home Page, United States Military Academy Department of Mathematical Sciences. http://www.dean.usma.edu/departments/math/courses/ma103/ ProspectiveStudents/CandidateWeb.htm

[4] *Class of 2015 to Enter West Point.* United States Military Academy News Release No. 21-11. June 21, 2011. http://www.usma.edu/dcomm/PressReleasesbd%5Cnr21-11_ class2015.html

[5] *Educating Future Army Officers for a Changing World: Operational Concept for the Intellectual Domain of the Cadet Leader Development System.* Office of the Dean, United States Military Academy, 2007 (p. 5). http://www.usma.edu/opa/clds/Educating %20Future%20Army%20Officers_Ver%203.pdf

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Effectiveness of Repeated Fundamentals Testing in Collegiate Calculus

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Abstract

Because fundamental mathematics skills are crucial to performing well in many collegiate level mathematics and science courses, these skills are assessed in one degree or another in most programs. Since 2009, West Point's integral calculus course has administered two fundamental skills exams, one each at the beginning and at the end of the semester. This paper describes our study of the fundamental skills program for integral calculus. The results suggest that retraining and repetitive testing can improve performance on fundamental skills tests. At the same time, the improvement in fundamental skills alone has a debatable role in terms of overall performance and retention of course wide learning objectives. However, a strong relationship does exist between fundamental skills proficiency and performance on comprehensive, end-of-course graded examinations.

Background

It seems to be a common plight of mathematics instructors that the students who enter their classrooms do not have the requisite mathematical skills necessary in order to master the material. Jourdan, et al. discuss this at length in their article which focuses on the skills that incoming collegiate students possess [1]. Additionally, in an article entitled Students' Difficulties in Calculus, David Tall discusses the difficulties that students of differential and integral calculus face. One of the problems that he discusses is the lack of algebraic manipulation skills that students have and that many of the basic skills that are often assumed "can no longer be taken for granted" [2]. Testing fundamental concepts, defined as the basic core mathematical competencies present in any typical high school pre-calculus course, is typically the goal of fundamentals examinations. An article discussing the perspectives of a number of experts in mathematics education regarding fundamental calculus knowledge identifies many of the items which are covered in the West Point fundamental concepts program [3].

The core mathematics program at the United States Military Academy consists of four courses: MA103: Introduction to Mathematical Modeling; MA104: Differential Calculus; MA205: Integral Calculus and Introduction to Differential Equations; and MA206: Probability and Statistics. In MA103, cadets take a fundamental concepts exam that covers 13 designated 'gateway' concepts, or fundamental skills, to all cadets in their first summer at West Point. These concepts include:

- Algebra and Real Numbers
- Algebraic Expressions
- Radicals and Exponents
- Factoring / Prime Numbers

- Linear Equations, Inequalities and Absolute Values
- Polynomials and Rational Inequalities
- Straight Lines
- Functions
- Quadratic Equations and Systems
- Trigonometric Functions
- Logarithmic and Exponential Functions
- Graphs and Graphing
- Analytic Geometry

In MA104, typically taken in the second semester of a cadet's first year at West Point, cadets take a Fundamental Derivatives Exam (FDE) that covers basic differentiation rules applied to single-variable functions: product rule, quotient rule, and chain rule. The Fundamental Calculus Exam (FCE) was introduced to MA205 to build on the foundation laid by the fundamental concepts program in MA103 and MA104 with one substantial difference. The FCE is a cumulative test, requiring cadets to demonstrate proficiency in gateway concepts from high school and MA103, differential calculus from MA104, and integral calculus from MA205. The breakdown of exam content is as follows: 25% gateway concepts; 25% differential calculus consisting of basic derivatives, chain rule, product rule and quotient rule; 50% integral calculus consisting of basic properties of integrals, five basic integrals that cadets must memorize, and integration using the substitution method. See Appendix A for a sample exam.

FCE Administration

The entire MA205 student population takes the FCE twice as a group: once in the third week of the course (FCE I) and once in the second to last week of the course (FCE II). The exam accounts for 5% of the student's overall grade each time it is taken. At one time, the exam was only administered during the start of the course. Beginning in the fall semester of Academic Year (AY) 2010, the FCE was administered at the beginning and end of the course with the hope that students would focus on and retain fundamental skills. Across all historical data, the exams consist of the same format and are of roughly the same difficulty.

In the period considered by this study, an 80% passing grade in the FCE was a requirement to successfully complete MA205 and advance to MA206. This meant that the exam administered early in the semester would be re-administered as many times as necessary to students who failed to achieve a passing grade of 80% until all cadets were successful. But suppose that a cadet numerically earned a passing overall grade in MA205 but never passed the FCE. If the claim is that FCE concepts are necessary for success in MA205, then the aforementioned scenario presents a problem.

In analyzing the FCE in the larger context of the objectives of the fundamental concepts program, we considered two questions. First, to what extent did cadets' FCE I scores at the beginning of the course relate to their FCE II scores at the end of the course? Second, to what extent does success on the FCE relate to overall success in MA205, as measured by performance on the final exam? In examining the first question, we use FCE I and FCE II data collected during the Fall 2010 and Spring 2011 semesters. In looking at the second question, we contrast scores on the MA205 comprehensive final exam, known as the Term End Examination (TEE), with the scores achieved on FCE I and FCE II. We chose to use the TEE score since it has the lowest amount of instructor-specific variability and because it covers all major concepts in the course.

Results and Discussion

Comparison of performance on FCE I and FCE II

Plots of individual scores from FCE I plotted against FCE II for the Fall of 2010 and Spring of 2011 are shown in Figure 1 and Figure 2. We examined the sets of scores broken down into four quadrants. The number of cadets in each quadrant and their associated final exam averages are presented in Table 1. The quadrants which are formed by dividing the *x* and *y* axes along the 80% passing mark are as follows:

- Quadrant 1: Students who failed to achieve an 80% on either exam (lower left)
- Quadrant 2: Students who passed the first administration of the FCE but failed the second (lower right)
- Quadrant 3: Students who failed the first administration but passed the second (upper left)
- Quadrant 4: Students who passed both FCE tests (upper right)

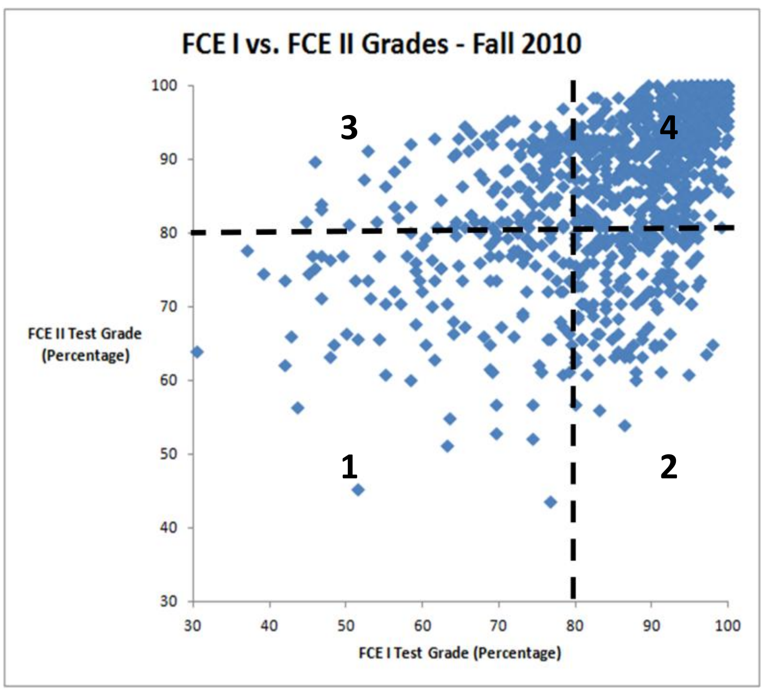

Figure 1: Fall 2010 Comparison

The most striking feature of the plots is the symmetry around the diagonal formed by the line FCE I Test Grade (Percentage) = FCE II Test Grade (Percentage). Across the entire MA205 student population, only 50.32% improved on their FCE I score when they took FCE II. We find it interesting that only slightly over half of the students did better considering that, by the administration of FCE II, there was already advance knowledge of the composition of the exam and a sizeable population of students had already taken it several times. In fact, for the overall student population, the probability that one does better on the subsequent exam is essentially the same as flipping a coin. A closer examination, however, does reveal differences in the population. However, if we condition on certain FCE I performances, we observe some intuitive results. For example, if we split the data and examine those students with an FCE I score of less than 80%, we find that 191 out of 227 cadets scored higher on FCE II in the Fall 2010 semester and 31 out of 42 cadets scored higher on FCE II in the Spring 2011 semester.

This improvement of scores on FCE II among the population of cadets initially failing FCE I makes sense when one considers that cadets scoring poorly on the first exam have more room to improve, and that all students in this population took *at least* one more administration of the FCE in order to achieve the 80% passing grade at some point between the two recorded scores. Another possible explanation is that cadets scoring poorly on the first exam have more incentive to better prepare for the second exam, especially given that the majority of course points have been assigned when FCE II occurs.

Conversely, if we condition on an FCE I score of more than 80% (Quadrants 2 and 4), subsequent grade improvement drops to 268 out of 686 cadets, or approximately 39% of individuals in the fall semester. For the spring semester, only four out of the 22 cadets scoring above 80% on FCE I improved on FCE II.

Applying similar reasoning, this data suggests that perhaps cadets who do well on FCE I fail to improve on FCE II because there is simply less room for improvement. It is conceivable that cadets who do well on FCE I prepare less for FCE II for some reason. Perhaps this is an indication that MA205 course content in general does not emphasize fundamental skills sufficiently in order to maintain proficiency throughout the semester. Is this an indictment of the subject matter in MA205 or could it be an indicator that we are not sufficiently discriminating in designating which skills we consider to be fundamental? In fact, even to discuss the need to embed fundamental skills into lesson content suggests that we ought to rethink what we are currently calling 'fundamental'.

Comparison of performance on FCE I/II and TEE

The second question that we examined was the relationship between performance on the FCE and performance on the Term End Examination (TEE, i.e., final examination). Based on our data, we observed that students who passed both FCEs with an 80% had the best overall final exam average, while those students who did not achieve an 80% on either graded event had the lowest average on the final. This is not especially interesting because good students tend to perform well on graded events in general. Therefore, it is not surprising that a certain population of students performs well on both the FCE and the TEE. Similarly, in is not especially interesting to observe that there exists a population of students that performs relatively poorly on both the FCE and TEE. Since we are curious as to the extent to which FCE performance relates to TEE score, we conditioned on FCE improvement in order to see if that provided any insight into TEE performance. Out of 930 cadets in the Fall 2010 semester, 468 improved from FCE I to FCE II. Those 468 cadets had an average TEE score of 82.37%. The 462 cadets whose score decreased between FCE I and FCE II had an average TEE score of 83.95%. If we further condition on cadets who passed both versions of the FCE, 277 out of 567 cadets improved. These 277 cadets had an average TEE score of 87.74%. The 290 out of 567 cadets who did not improve have an average TEE score of 87.50%. Finally, we looked at the subset of cadets who scored less than 80% on both FCE tests. Of that group of 106 cadets, 70 improved from FCE I to FCE II. These 70 cadets earned an average score of 70.07% on the TEE while the remaining 36 cadets scored an average of 73.03% on the TEE.

The data shows that improvement on the FCE has no statistically significant impact on TEE score. When performing a two-sample *t*-test to determine if the mean TEE score for cadets improving on the FCE is different from the mean TEE score for cadets not improving on the FCE, with our null hypothesis being that the means are identical, the associated *p*-values provide marginal support for the alternate hypothesis only when looking at the particular subset of the student population that passed one FCE and failed the other. When examining the subsets of the population that passed both FCEs and failed both FCEs, there is insufficient evidence to reject our null hypothesis.

Table 1: Fall 2010 TEE performance for students based on previous FCE performance

Group	Number Improving	Mean TEE Score	Number Regressing	Mean TEE Score	p-value
Pass both FCE (Quadrant 4)	277	87.74%	290	87.50%	0.7284
Fail both FCE (Quadrant 1)	70	70.07%	36	73.03%	0.1634
Pass one FCE (Quadrants 2 & 3)	121 (Quadrant 3)	77.21%	136 (Quadrant 2)	79.27%	0.0715

Of note is the population of students who only achieved an 80% on one of the FCEs. The cadets who failed FCE I but passed FCE II had slightly lower TEE scores. Those students who find themselves in this quadrant were required to take multiple versions of the FCE and receive remedial training until they were able to pass the FCE with an 80%. In spite of the additional instruction and training, however, the TEE percentage was still lower than those who passed the first FCE but failed the second. This hints that superficial learning is being used to achieve a test score of 80%, but a long term understanding of the fundamentals is lacking.

Further Work and Conclusions

We observed noticeable improvement between FCE I and FCE II among the population of students who initially failed to achieve a passing grade on FCE I. This group of students received remedial training and as many opportunities to pass an FCE as needed before the final administration of FCE II. This suggests that retraining and reinforcement through repetition can produce significant improvement in test scores. Additionally, this improvement occurred among a student population that is on the lower end of the grade spectrum.

Regarding the predictive ability of FCE performance on TEE performance, our data did not support any appreciable differences between the populations that we compared, specifically those cadets who improved on FCE II versus those who did not. There does seem to be a connection between students who score well on the FCE and performance on the final exam, though we are unsure whether this performance reflects underlying study patterns or inherent skill which makes individuals proficient at fundamental skills.

In the fall semester of 2011, we adjusted the FCE administration by only allowing two retests for the first administration of the exam with a full grade replacement. We hope that this change will encourage better internalization of fundamental concepts, while putting a positive incentive on students to learn the material. The results of this change will be analyzed in the future.

Repeated focus on fundamental skills is important since an improvement was seen in subsequent testing by a large portion of individuals who took multiple administrations of the fundamentals exam. Repeated testing of fundamental skills from the start of the semester until attaining a demonstrable level of proficiency helped a sizable number of the course population with the administration of FCE II. Finally, the performance of those students who failed both FCEs seems to mimic that of superficial learning, with no discernable pattern. Again we find ourselves with the proverbial question of how to encourage deep learning and self-learning on the part of our students.

References

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Appendix A (Sample Fundamental Calculus Exam)

MA 205 11-01 PAGE – 1
Fundamental Calculus Exam Version 1
Total Weight: 125 points

Cadet______________________
Section_____________________
Sep 10

MA 205 11-01 Fundamental Calculus Exam

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING WORK.

1. You may use your issued calculator on this quiz.

2. This quiz consists of 5 pages including the cover sheet.

3. Sufficient work is required to indicate the method of reasoning and the operations performed. SHOW ALL WORK. Clearly indicate your final answer.

4. All work written will be graded unless marked through or explicitly marked with words to the effect of "do not grade."

5. Work only on the front side of a sheet of paper. If you need more space, use a separate sheet for each problem continued. Clearly indicate which problem is continued at the top of the sheet along with your name.

6. Always use proper mathematical notation.

7. Assume all angles are in radians.

7. You have 45 minutes to complete the exam.

Problem	Total Points	Score
Page 1 Gateway Concepts	20	
Page 2 Derivatives	20	
Page 3 Integral Properties	40	
Page 4 Substitution	45	
Score	125	

Appendix A (Sample Fundamental Calculus Exam)

MA 205 11-01 PAGE – 2
Fundamental Calculus Exam Version 1
Total Weight: 125 points

Cadet______________________
Section_____________________
Sep 10

1) (5 Points) Solve for all values of x that satisfy the equation: $x^2 - x - 7 = 0$.

2) (5 Points) Simplify the expression $\left(\dfrac{6x^4 y}{3x^{\frac{1}{2}} y^5}\right)^2$.

3) (5 Points) Solve $|5x+2| > 7$ for all values of x.

4) (5 Points) Find the solution to the following system of linear equations.

$$x + y = 3$$
$$-x - \frac{3}{4}y = -4$$

Appendix A (Sample Fundamental Calculus Exam)

MA 205 11-01 PAGE – 3
Fundamental Calculus Exam Version 1
Total Weight: 125 points

Cadet____________________
Section____________________
Sep 10

Find the indicated derivatives.

5) (5 Points) $v(x) = 10x^3 + \frac{1}{x^2} + \sin(x)$, find $v'(x)$

6) (5 Points) $y(x) = 6\ln(4x-2)$, find $y'(x)$

7) (5 Points) $f(x) = 3e^{-2x}\cos(x)$, find $f'(x)$

8) (5 Points) $h(x) = \frac{(2x^3+1)^5}{(x^{-2}-5)}$, find $\frac{dh}{dx}$

Appendix A (Sample Fundamental Calculus Exam)

MA 205 11-01 PAGE – 4
Fundamental Calculus Exam Version 1
Total Weight: 125 points

Cadet____________________
Section____________________
Sep 10

9) (10 Points) Evaluate the Integrals:

(a) $\int e^x \, dx$

(b) $\int x^n \, dx, where\ n \neq -1$

(c) $\int \sin(\theta) d\theta$

(d) $\int \cos(\theta) d\theta$

(e) $\int \frac{1}{t} dt$

10) (15 Points) Evaluate the Integral: $\int \left(\frac{e^x}{5} - \frac{1}{x^5} + 10x^{-1} + \cos(x) \right) dx$

11) (15 Points) **Write as a single integral** in the form $\int_a^b f(x)\,dx$:

$\int_{-10}^{-5} f(x)\,dx + \int_{-5}^{2} f(x)\,dx - \int_{0}^{2} f(x)\,dx$

Appendix A (Sample Fundamental Calculus Exam)

MA 205 11-01 PAGE – 5 Cadet____________________
Fundamental Calculus Exam Version 1 Section____________________
Total Weight: 125 points Sep 10

12) (15 Points) Evaluate the Integral: $\int_0^{\frac{\pi}{2}} 10\sin(\theta)\cos^2(\theta)\,d\theta$

13) (15 Points) Evaluate the Integral: $\int 5\left(2x+3x^2\right)\left(5+x^3+x^2\right)^4 dx$

14) (15 Points) Evaluate the Integral: $\int \left(6x^2+x^{-1}\right)e^{2x^3+\ln(x)}\,dx$

Fundamental Skills: To learn or not to learn?

Dr. Chris Arney
Department of Mathematical Sciences
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Have you ever watched a basketball team warm-up before the game? What can you tell about their abilities as players and as a team, if anything? Sometimes you can see very clearly their advanced skill level -- crisp passes, quick-release shooting, sky-high rebounding, slam dunks, and fancy moves. Sometimes you can see basic skills being displayed and reinforced -- concentrated proper-form foul shots, perfect lay-ups, excellent positioning, and organized movement with a purpose. And other times, you can see the coordinated teamwork emerge -- organized drills, encouraging spirit and communication, precise group drills, and a cooperative attitude. Of course, excellent teams have some of each of these attributes, but the real skills of the players and team aren't fully revealed until the game starts. During the game, not every team or every player matches my warm-up assessment. Amazing to me is that some players are able to demonstrate and perform the advanced skills and coordinated teamwork, even if they do not possess the basic skills. It is surprising to me that some players are able to make highly athletic and amazing plays, but are poor shooters (especially foul shots) and poor passers or defenders. I am sure the coaches of players like this are frustrated, just as I am with the students in my class, when they do not possess, develop, or refine their basic fundamental skills. But, like in basketball, all is not lost -- these basic skills just might not be the prerequisite to success that I once thought they were. Great basketball players, especially many in this modern, post-Jordan era, have poor basic and fundamental skills. And I suspect that many good students, in this information age, are able to perform high-level modeling and problem solving despite not having all the basic algebra-trig skills we desire they possess. Likewise sometimes a player doesn't appear to have the same athletic level (poorer basic and/or advanced skills) as the others and yet when the game is played turns out to be the star of the team. The player organizes the other players, enables the others, cooperates, assists, and eventually leads the team. Are some of my lesser skilled students learning to become those kind of team leaders for the game of real life?

In basketball, it is very likely that the skills are more transparent than classroom mathematics skills. And the big difference between sports coaching and our classroom teaching is that we never really see our students play their "game". Education is just the practice, warm-up mode for their real-life games that occur long after students leave our classrooms. And the substantial difference is that one trains to play sports, but mathematics is education that is meant to stay with students forever and enable them to think in entirely new and higher-levels than we are able to teach them.

So what is my point? Just like a good, John Wooden-like coach, I believe as conscientious math educators we need to educate students at all levels of mathematics (for lack of better words -- basic skills, advanced skills, and teamwork). We need curricula and assessments, and pedagogy that keep all these levels in balance. I think we need to drill some of the skills into our students, but don't drill and kill or think of these as prerequisite skills. I think we need to develop higher-level thinking, modeling, problem-solving skills, but don't think these are all you need to do or that they come easily to students -- this is difficult stuff. And finally, we need to develop their ability to communicate, cooperate and collaborate. After all, a player with all the natural skills in the world, but who can't run the plays or fit into the team will never help a team win games. However, in basketball, a player may still entertain the crowd --- a factor in sports that we don't need to develop in our mathematics students.

How do students learn? I am not an educational psychologist, but I am aware of several different perspectives or theories of learning -- behaviorism, cognitivism, developmentalism, and constructivism. Let me contrast the ends of this educational spectrum through my perspective. From a behaviorism viewpoint, the learner just learns more information (obtains a greater and greater body of knowledge). In a way, this is a perspective that supports breadth -- learning doesn't really build upon itself, there are just more and more facts to learn to become smart. In the constructivism/cognitivism theory, students use their memory structures and their traits, beliefs, motivations, and emotions to determine how information is perceived, processed, and stored in a form of instructional scaffolding. This theory supports a depth perspective where there is a mental structure being built so that later information builds upon earlier learning. I think many mathematics educators are constructivists who see a mental pyramid being built in our students' minds that allows more advanced mental attributes lying on top of the basic skills until the zenith of the pyramid is their deepest, most complex, reasoning about mathematics. Of course, everyone has holes and weak parts in their pyramid, but if there is a good enough foundation from the earlier courses, we try to get our students to build a higher, wider, and sounder structure to their pyramid. Unfortunately, if there are too many holes or weaknesses, we have trouble knowing how to teach our students the higher level topics and ideas until they shore up their foundation. Many times, I have found bright students who had gone into short-term memorization mode in their math courses. I could sense that their structure was decaying instead of strengthening. I desperately wanted to go back to 7th grade topics and shore up the foundation, but there was never the time to do that monumental task.

So again, I come back to balance. If it is something like a pyramid that is being constructed in our students' minds, then we have the obligation to give them the opportunity to shore up their weaker foundation areas by giving them fundamental skills and the equivalent obligation to build onto the top layers through more advanced topics so they have a richer, larger, stronger, better structure called "their own mathematics."

What are our students learning? This is an easy one to answer -- calculus. Well, we also need to add calculus' close relatives -- differential equations, analysis, numerical computing, optimization, and linear algebra. In many ways, the K-12 math curriculum is based on enabling students to reach calculus during senior year in high school or first year in college. Although I am happy to say at USMA, there is a bit more than the typical calculus family in our required curriculum: probability, statistics, discrete mathematics, and lots of modeling for all our cadets, and options for graph theory and more algebra for students who study math in depth. But much to my chagrin, there is very little geometry in school or college these days. But why is calculus such a big player in 21st century American math education? Frankly, I haven't a clue. I understand that calculus was important in the 19th century as society was trying to build its industrial infrastructure -- engineering advancement was essential and we needed to use math and the physical sciences to advance our fledging technologies. But by the time we entered the information age in the late 20th century, our priorities were no longer engineering and physical science as we needed to advance the information, computing, and social sciences that dominated our highest priority issues and problems. Yet, we haven't changed our math education very much. Such a change is difficult -- we are set in our ways. But by now shouldn't we have our students taking more discrete math, graph theory, game theory, combinatorics, geometry, and matrix algebra in high school and college and end our calculus obsession? Don't get me wrong, I love calculus -- it is beautiful and powerful and the language of engineering and physical science, but it really isn't necessary or preferred for most professions and college-educated people. It is not even necessary for our undergraduate math majors -- just a nice branch of mathematics to see continuity and dynamics. Yet we have an entire K-16 curriculum designed for the 1% of the people in our country who will be engineers or physical scientists who really need calculus. At USMA, perhaps the percent rises to 10%. By the way, at USMA, we do have several fantastic math/interdisciplinary application courses in cryptology, chaos, fractals, complexity, energy management, climate change, health care policy, operations research, information science, network science, and computing that really excite students and enrich our cadets' education. I believe we need more of our students (I would like to say all of them) to experience this kind of math excitement.

I will end with a suggestion you probably expect by now -- balance. Let's balance the skills we teach (basic, advanced, and teamwork) in not just calculus and functions, but in the many areas and structures of mathematics. While one can see something noble in the perseverance of a student trying to master the fundamental skills of algebra through his or her 5th or 6th time through a course in the subject, I would rather see that student taking an appropriate level geometry or discrete math class and building a perspective and skill level in that exciting area of mathematics. And to emphasize another point in the idea of balance, we need to ensure all our courses and programs have communication, cooperation, and teamwork associated with their content goals so we produce the leaders we need to solve problems and build our future society.

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