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CONTENTS

A Note from the Editor

This issue features two interesting articles within the theme *Beyond the Core Math Program: Math Electives that Service Engineering Disciplines*, one from a required USMA service course and another from a USMA elective service course. We are also glad to share a thought-provoking analysis of USAFA endeavors to complement face-to-face Extra Instruction (EI) with EI Videos to help struggling students. If you find you have something to add to this discourse, please consider submitting a follow-up or response for the Fall 2012 issue.

The upcoming Fall 2012 issue will emphasize the topic-- *Technology in the Probability & Statistics Classroom*. How has cadets’ use of technology evolved in your probability and statistics courses, and where is it going next? To what extent are technology-oriented labs and problems used to support conceptual learning in your department? Are computers allowed (or necessary) on exams? If so, under what restrictions?

As always, we also continue to welcome general topic papers you wish to submit. The Editorial Policy is included at the back of this bulletin. October 10, 2012 is the deadline for submissions for the Fall issue. We look forward to hearing from you!

Brian J. Lunday
Co-Editor-in-Chief
brian.lunday@usma.edu

Mathematics: The Language of Engineering
A Discussion of Teaching Mathematics to Engineers

Major Benjamin Gatzke
Department of Mathematical Sciences
United States Military Academy

Abstract

Engineers and mathematicians alike are aware of how crucial the understanding of mathematics is for success in an engineering curriculum. This awareness drives the sensitive nature with which the Department of Mathematical Sciences at the United States Military Academy (USMA) develops its Engineering Mathematics course. This course teaches four different engineering disciplines (Mechanical, Civil, Electrical, and Nuclear), each with different accreditation requirements to fulfill. The myriad of demands results in a rigorous syllabus of topics covered over a three credit hour course. Meeting these demands successfully requires a balance of mathematical theory and application as well as steady communication between our department and the leaders of each engineering program directorate. Our intent is that these two requirements work hand-in-hand and drive careful, subtle changes to the course structure to allow fluid improvements which provide the best possible mathematical background for the success of engineering students as they complete a demanding curriculum within their own major. This paper will discuss course content and conduct in the classroom, evaluation of teaching methods used in Engineering Mathematics, communication between our department and parent engineering programs, and challenges we face. The intent of this discussion is two-fold: 1) To provide insight to other service academies into how we conduct our Engineering Mathematics course and 2) To establish a communication link with the other service academies which will hopefully lead to a network of discussions to facilitate improvement of Engineering Mathematics course conduct in general.

Introduction

Since its establishment by Congress in 1802, the United States Military Academy has focused its academic curriculum heavily on engineering and the mathematical sciences. [1] Although the number of academic disciplines and majors available to cadets has grown substantially over the past 209 years, the focus on Engineering and science has remained a steadfast cornerstone of West Point education. A critical task, to support the engineering departments, is to provide the conceptual foundations of mathematics to those brave cadets who covet the title, upon graduation from West Point, of an Engineer.

Our Engineering Mathematics course, MA364, focuses on four particular groups of engineering students: Civil, Mechanical, Electrical, and Nuclear. Usually, we attract a few students from other majors who take the course as an elective (i.e. Physics, Information Technology). Figure 1 below shows our student population for the last two years.

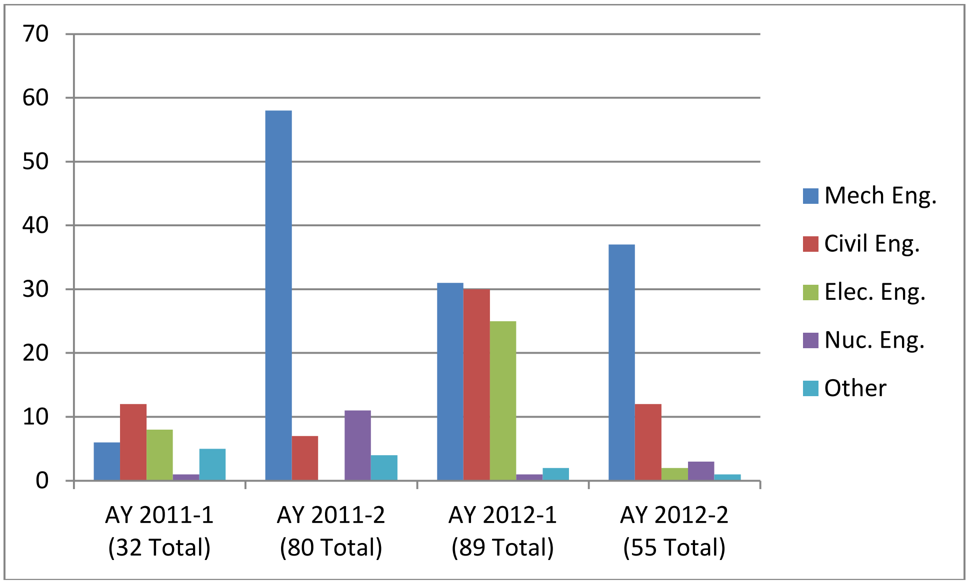

Figure 1. MA 364 Student Population Composition for AY 2011 and AY 2012

Course Content

The purpose of this course is to provide additional mathematical techniques and deepen the understanding of concepts in mathematics to support continued study in science and engineering. [2] It is crucial to accomplish this in a manner which is manageable for students while accomplishing more than mere surface learning. For instance, it is preferable for students to understand how a surface integral is derived prior to learning how to use the equation to calculate Flux through a surface. Students completing MA364 should understand how and why certain mathematical concepts work, not just that they work.

According to Dennis Zill and Warren Wright, co-authors of the text Advanced Engineering Mathematics, which we use as the text for our course, a course in engineering mathematics differs from a standard calculus or differential equations course since it is less standardized and sometimes varies considerably between two different academic institutions. [3] Therefore, MA364 is divided into three blocks which inherently, due to their vast differences, become three micro-courses in Vector Calculus; Ordinary Differential Equations and an Introduction to Functions in the Complex Plane; and Introduction to Partial Differential Equations. A complete list of topics is included as Appendix A.

Based on feedback from our engineering programs and regular self-assessments, we evaluate and modify our graded events from semester to semester (MA364 operates in both Fall and Spring). Last Fall (2011), we experimented with a discipline-specific project. In the current term, students are required to assess, both written and through presentation to the class, a scholarly journal article of their choosing involving the application of mathematics. Our graded event scheme from the current semester includes two mid-term examinations, one final examination, seven problem sets (including one focused on reviewing fundamental skills), and one scholarly journal article review and briefing. Although demanding, students are able to fulfill these course requirements despite significant requirements associated with their selected major.

Deliberate Evolution of Course Content

Feedback and assessment of relevant competencies necessitate a continuous evolution of the content and syllabus of MA364. Generally, from semester to semester, changes are minor and designed to streamline the path to our desired course outcome. These changes are driven by a desire to improve the ability of our students to connect mathematics with engineering. A consistent, two-way communication between our department and the Civil, Mechanical, Electrical, and Nuclear Engineering Program directorates allows us to continuously refine our course into what all interested parties view as the proper blend of rigorous mathematics and applications to relevant problems.

The Department of Mathematical Sciences has official liaisons with the other science and engineering departments to ensure that math course curriculum guidelines are met for formal purposes, such as ABET accreditation and fulfilling academic intents set forth by the Office of the Dean. In addition, we feel that an informal synchronization for math courses with their respective engineering programs is also essential for streamlined continuity. In the summer, we plan a series of meetings between each Engineering Program Director, the MA364 Course Director, and the Math Department Electives Program Director with any other key players to discuss the upcoming MA364 course syllabus and major graded events. This meeting is intended as a two-way discussion, and we solicit feedback and recommendations from each program, using both historical data and our upcoming plan, to nest our desired outcomes with those of each engineering program. For example, this past summer, we introduced the idea of a discipline-specific course project which required students, in pairs, to apply ODEs to a realistic problem within their specific major, given as little guidance as possible. The departments were all optimistic, so we integrated the course project into MA364 for the first time in the course's recent history. The project effectively allowed students to bridge the concepts encountered in MA364 with a problem they might see in the future . The discussion with each engineering program served two purposes in the evolution of the project: 1) The programs all were aware that we were integrating a project, and 2) We knew from the discussion that they supported the idea. As we move forward, we will continue to actively pursue these open lines of communication to effectively balance our curriculum with the needs of our engineering counterparts.

Evaluation of Classroom Procedures

Since 1817, when Sylvanus Thayer assumed the duties as Superintendent of USMA and instituted significant fundamental changes to academic curricula and graduation standards, varying degrees of the Thayer Method have existed in our classrooms. This method expects students to assume responsibility for their own learning and to attend class having studied the lesson and completed assigned homework for that particular day. [4] In Engineering Mathematics, we have partially instituted this method. Due to the breadth of topics covered, we expect students to review the material prior to class and attempt some of the suggested homework problems, if they understand the material. We then expect them to complete the suggested problems after the lesson is taught. Questions on the homework problems are then answered to begin the following lesson. This sequence puts some responsibility on the students to prepare for class but allows them to address challenges faced in understanding new material without placing the section behind the desired pace.

The amount of course material that we must cover in 40 lessons demands a constant reflection upon our balance of theoretical knowledge versus application of topics to solve problems. We alleviate this conflict, to some degree, by analyzing each topic covered and selecting the critical few which engineering students must understand more theoretically in order to sufficiently apply them to realistic engineering problems. For instance, it is difficult for a Mechanical Engineering student to apply a 2^{nd} Order ODE in developing an appropriate shock absorber model without first understanding and deriving each component of the non-homogeneous spring mass equation. However, explaining the theoretical reasoning behind why one can use the Dirac Delta function to represent an impulse forcing function, even though it cannot be truly defined as a function, is not really a necessary discussion for engineering application. We tend to have more theoretical discussions in the classroom on the following topics:

1. Derivation of the Line Integral Equation
2. Derivation of the Surface Integral Equation
3. Development of the three cases of 2^{nd} Order ODE Solutions
4. Derivation and visualization of Spring Mass Solutions
5. Derivation of the Laplace Transform
6. Showing Orthogonality of Sets of Functions
7. Development of the Fourier Series Approximation for a Function
8. Development of a PDE Solution.

Instructor and Electives Program assessments and student feedback have shown that our current approach is balancing theory and application quite well and leading students to success in MA364 and their follow-on engineering courses.

Another common theme across MA364 is familiarization of students with the use of their textbook. As mentioned before, we explore some topics in great theoretical detail while we do not with others. This forces students to use their textbook to explore certain theorems, proofs, and examples on their own to further understanding of the course material. As experienced cadets and future engineers, the expectation is that they can manage these requirements without issue. Furthermore, we allow the text as an exam reference but do not allow them any outside notes. Any annotations made inside of the book are considered an authorized exam reference. When asking students their thoughts, most actually prefer the use of the book rather than outside notes. They express extreme disdain though when we take away their calculators for exams! However, our expectation is for them to know that $sin\left(\frac{\pi}{2}\right) = 1$, and we reassure them that $3e^5 + \ln(3)$ is a perfectly suitable answer – and that the approximation of *446.3381* provided by their calculators is actually an inexact answer! Overall, students perform very well on exams with the references provided.

An enduring trait for MA364 is that it remains, for the most part, a "stubby pencil" course. This means that we have integrated very little technologically aided student-level problem solving. Almost everything our students complete is by hand. Instructors will use computer programs, mainly Mathematica, to visualize certain products to aid in students' conceptual understanding of topics. Some examples of items we might visualize for them in class are vector fields, three dimensional surfaces, differential equation solution curves, Fourier Series graphs, and PDE solutions. Students are also asked, on occasion, to plot solutions as part of their graded problem sets. These problem sets are designed to challenge them above the level of suggested problems and exams, so the assistance of computer programs is sometimes required to achieve this goal. Everything on exams and quizzes is done by hand; this means that students must thoroughly understand the problems they attempt in order to achieve success on in-class graded events. This does not imply that we do not embrace technology; the thought here is that students must possess the capability to understand and execute the basic techniques, rigorously, in order for technological solution and visualization techniques to hold useful meaning. This tactic remains effective, and we plan to continue in such a manner for the foreseeable future.

Challenges

In addition to the difficulties faced with the pace and demanding syllabus of MA364, two additional challenges require acute attention to ensure our students' ability to attain the desired outcomes of the course: 1) Competing courses and their extensive academic requirements and 2) A mathematically diverse student population. Our ability to manage these three challenges is critical to the continuing improvement of MA364 to its respective engineering programs.

In the demanding academic environment of our students, it is common for many of them to have three or four problems sets due during one academic week – each requiring several hours of work to complete. Since their other classes are mostly within their major, it is easy for our coursework to become low on the priority list. Therefore, a degree of balance is required. We must continue to challenge them through problem sets without committing them to several hours of outside work, and we must attach a point value to such assignments to warrant an effort rivaling that of assignments for other courses. We have modified our out of class assignment scheme quite significantly over the past year in an attempt to attain this goal.

In Spring AY 2010-2011, there were four out of class problem sets at 50 points each (the four together worth a total of 10% of the course grade). The problem sets were long and tedious in places, requiring an average of four to six hours each for students to complete. The top students had no issues with balancing these assignments and their other academic demands, but the mid to lower level students struggled and often times submitted poor assignments. They could get away with this due to the low point value of the assignments. In the current semester, one year later, we have incorporated one scholarly journal article review, requiring both a written and oral presentation, worth 100 points, and seven problem sets worth a total of 500 points (out of class assignments totaling 30% of the course grade). The assignments are now more frequent but shorter in nature. The problems are still challenging, yet the work is less tedious. Now, on average, a problem set requires two to three hours of work. Since, for either graded event scheme, students tend to wait until a few days from the due date to begin work, they are less inclined to become frustrated and more inclined to turn in a solid product. More importantly, they are more inclined to learn the desired course objectives. Overall, students spend the same amount of hours on out of class assignments, but they are sequenced in such a way that they are more effective for learning in our course while allowing us to compete less for out of class time with courses in their major.

The mathematical diversity of our student population presents another challenge; our students measure differently historically, ranging from A+/A+ in our advanced core program (Multivariable Calculus, Mathematical Modeling, and Differential Equations) to a C+ average in our standard core program (Mathematical Modeling and Calculus I / II). This can pose a challenge for each instructor to find a level on which to focus his teaching efforts. Instructors much ensure that they have enough in-class material to keep the upper level students occupied since the cadets will inevitably work at different paces during board work. Encouraging the stronger students to help those struggling with a certain topic and pairing these students together to work through in-class requirements are also effective tactics for bridging gaps in ability. Overall, working through this challenge lies with individual instructors who must assess their sections at the beginning of each semester and adjust their teaching tactics accordingly, since sections can be vastly different from one another.

A Suggestion for Future Refinement

Our students tend to have anywhere from one semester to four semesters between their last applied mathematics course and MA364. As with any skill, mathematics fades with absence. Additionally, those students who take Engineering Mathematics during the second semester of their third year or during their fourth year are already deep into their engineering curriculum, so the course becomes, parallel at best, or sometimes obsolete. This is because MA364 is designed as a pre-requisite for certain engineering courses; if students are taking these courses concurrently or have already taken them before taking MA364, it defeats the purpose of the course. One way to mitigate both of these challenges is to offer Engineering Mathematics only as a course available during second semester of students' second year. This course of action is feasible, since everyone has declared their major by the beginning of their fourth semester. Also, students would have, at most, a one semester break from applied mathematics, and they should be on the very forward edge of their respective engineering curriculum at this point. Therefore, MA364 will act as intended – as a true pre-requisite course to classes in the Mechanical, Civil, Electrical, and Nuclear Engineering disciplines. Such a course of action would have to be agreed upon by all of our counterparts in the supported engineering programs. The projected mathematics curriculum for both advanced and standard core students is shown in the tables below. The courses referenced below are as follows: MA103: Mathematical Modeling and Introduction to Calculus; MA104: Calculus I; MA205: Calculus II; MA153: Advanced Multivariable Calculus; MA255: Mathematical Modeling and Introduction to Differential Equations; MA206: Probability and Statistics.

Regular Core Sequence				
1st Semester	2nd Semester	3rd Semester	4th Semester	5th Semester
MA 103	MA 104	MA 205	MA 364	MA 206
Advanced Core Sequence				
1st Semester	2nd Semester	3rd Semester	4th Semester	5th Semester
MA 153	MA 255	MA 206	MA 364	N/A

Conclusion

Despite the challenges facing MA364 as a service course to Mechanical, Civil, Electrical, and Nuclear Engineering majors, we continue a fluid approach to course management to ensure that our students receive the best mathematical background possible which will inevitably facilitate their success as engineers. Over the past year and a half, student feedback and information obtained from the respective engineering program directors have led us to believe that students taking Engineering Mathematics are provided with the mathematical concepts and techniques to be successful engineering students. Continuing efforts to solicit as much feedback as possible from all sources and to cross talk with all supported engineering programs should continue to shape Engineering Mathematics into an even better course well into the future.

As stated before, MA364 is not a standardized course and, therefore, it requires fluid improvements to enhance the learning experience of its students. Hopefully this paper will extend the breadth of course evolution outside of the walls of USMA to our sister service academies and, perhaps, to other universities as well. There is undoubtedly a wealth of knowledge that, collectively, can lead us even further toward the primary goal of providing the best mathematical foundation possible for undergraduate engineering students.

Endnotes

[1] Amy E. Shell, "The Thayer Method of Instruction at the United States Military Academy: A Modest History and A Modern Personal Account" In: *PRIMUS (Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 12:1, 27-38.

[2] This information was obtained directly from the USMA Redbook Course Description for *MA364, Engineering Mathematics*.

[3] Zill, Dennis G. and Warren S. Wright, Advanced Engineering Mathematics, Fourth Edition, Jones and Bartlett Publishers, 2011, pg. vii.

[4] Shell, 27-38.

Appendix A: Detailed Topic Breakdown for MA364

I. Vector Calculus *(14 Lessons)*
   a. Review of Vector Functions, Scalar Fields, and Vector Fields
   b. Gradient, Curl, and Divergence
   c. Line Integration (Applied to Work done by a vector field along a curve)
   d. Path Independence and the Fundamental Theorem of Line Integrals
   e. Green's Theorem in the Plane
   f. Surfaces and Surface Integration (Applied to Flux of a vector field through a surface)
   g. Stokes' Theorem
   h. Triple Integration and Gauss' Divergence Theorem

II. Ordinary Differential Equations (ODEs) and an Introduction to the Complex Plane *(16 Lessons)*
   a. Review of Homogeneous 2nd Order Linear ODEs
   b. Non-Homogeneous Linear ODEs (Method of Undetermined Coefficients)
   c. Spring Mass Systems
   d. Operations on Functions in the Complex Plane
      i. Addition, Subtraction, Multiplication, Division, Conversion to Polar Coordinates
      ii. Calculation of Powers and Roots

iii. Euler’s Formula and Exponential Functions

e. Laplace Transforms
f. Inverse Laplace Transforms and Transforms of Derivatives
g. Translation Theorems
h. Unit Step (Heaviside) and Impulse (Dirac Delta) Forcing Functions

III. Introduction to Partial Differential Equations *(10 Lessons)*

a. Orthogonal Functions
b. Functions Expansions using Fourier Series
c. Fourier Series using Half-Range Cosine and Sine Series Expansions
d. Introduction to PDEs (Separation of Variables Method)
e. Modeling and Solving the 1-D Heat Equation (and 1-D Wave Equation, if time permits).

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A New Course for a New Mission: Mathematics for Space Applications

Dr. Jessica Libertini
Department of Mathematical Sciences
United States Military Academy

Abstract

In support of the new Space and Missile Defense Research and Analysis Center hosted at USMA, the Mathematical Sciences Department developed a new course designed to introduce a variety of mathematical concepts in the context of space operations applications. This new course, *Mathematics for Space Applications*, is the first of its kind. It is neither like a preparatory math course which may teach similar mathematical background skills divorced from application, nor like an introductory astrodynamics course which merely applies a memorized set of equations without exploring the underlying mathematical principles and derivations. Not only is the course content unique, but the students encounter the material in a non-traditional format through the development of a peer-taught learning community for a majority of the material coupled with individual in-depth research tailored to each cadet's interests. This paper explores the background behind the addition of this course, the curriculum covered by the course, the peer-taught learning environment, and the individual research component of the course.

Key words: astrodynamics, course development, learning community

Background

In 2008, West Point and the Space & Missile Defense Command (SMDC) began outlining a collaborative relationship that would ultimately birth a new research and analysis center, spur a rejuvenation of our cadet astronomy club, and promote the development of three space-related academic courses available to our cadets. These three courses are: a newly revamped *Space Physics* course taught by the Physics Department; a new course taught by the Electrical Engineering and Computer Science Department in *Satellite System Design* in which a cube satellite is designed, built, and tested over a three year cycle; a new course taught by the Mathematical Sciences Department covering the *Mathematics for Space Applications* in which a variety of mathematical concepts are presented in context of astrodynamics related applications. A key goal of these three courses is to provide interested cadets with an opportunity to learn about Army Space Operations during their time as cadets.

The mathematics course is designed to provide both a more in-depth study of the equations used in orbital mechanics and an opportunity to explore additional mathematical concepts that would not present themselves in a traditional orbital mechanics course. Some of the classical topics addressed include gravitational theory, conic sections, conservation laws, the classical results for the two-body problem, and a thorough treatment of coordinate systems and reference frames. The non-traditional topics include topology and geometry of the universe both locally (relativistic effects) and globally (large scale shape of space), as well as numerical methods for the study of orbital perturbations due to external forces.

The author was selected to develop this course due to her prior industry experience with satellite systems for supporting the Missile Defense Agency. During the initial development of the course, the author coordinated with officers from FA40 (Army Space Operations Functional Area), faculty from USAFA's Aero/Astro program, websites for undergraduate and graduate courses in astrodynamics, and practicing astrodynamicists. It quickly became apparent that there was no existing course that attempted to present the higher level mathematics in the context of astrodynamics applications at an undergraduate level. In fact, it is far more common for a "space math" course to simply be a prerequisite mathematics course for engineering (possibly aero/astro) majors that covers foundational topics, such as coordinate transformations, divorced from actual applications. While it is certainly important to service these majors with mathematical background, these topics can be motivated by being taught in context, and that was the motivation for developing this unique course. Therefore, after collecting a few key ideas from undergraduate astrodynamics courses, the work of realizing this new course began.

Curriculum

The course is broken into three "blocks" each of which covers an almost self-contained group of topics. The first block begins with gravitational theory. The cadets explore the gravitational fields of oddly shaped planets of unusual density profiles, and they learn to justify the point-mass assumptions of classical orbital mechanics through analyzing the gravitational field of spherical shells. From there, we explore the canonical two-body problem, deriving the trajectory equation. In addition to solving the equation analytically, we immediately introduce some basic numerical techniques to solve this vector ODE, verifying our analytic results by plotting the numerically calculated orbits showing circles, ellipses, parabolas, and hyperbolas. After learning about conserved quantities and deriving the relevant equations, cadets are able to verify their findings about open and closed orbits using their numerical codes. After exploring the two-body problem, Kepler's Laws, and Newton's Laws, we move on to analyze Kepler's problem and develop Kepler's equation. At this point, we are able to explore a variety of mathematical techniques to solve Kepler's equation. The last portion of this block is spent learning about the Classical Orbital Elements (COEs) and developing a deep intuitive understanding of the equations used to convert between COEs and a set of position and velocity vectors. We also discuss an orbit's COEs can be optimized to support a particular mission.

The second block starts with a thorough treatment of reference frames and spatial coordinate systems as well as an introduction to the accuracy of time systems and the impact of relativistic effects on temporal calculations. While some of this work is done divorced from application, the focus is quickly returned to space operations through the development of algorithms to transform between satellite-based coordinate systems and geo-based coordinate systems. We further develop these algorithms to account for viewing the satellite from a ground-station affixed to the surface of a rotating Earth. With the foundation laid, we derive the necessary calculations to perform Earth observations. Based on these, we look at orbit determination from a historical perspective. This problem of calculating the orbit of a celestial object based on a finite discrete set of angle-based observations attracted some of the greatest mathematicians of all time, including Laplace and Gauss. Using these results, we are able to calculate the orbital elements of an observed object and then, coupled with results from the first block, we are able to predict future observations. Naturally, all of these calculations rely on the simplifying assumptions of the two-body problem, and as any practicing astrodynamicist knows, these assumptions are far from accurate for geocentric satellites. Therefore, at this point in the course, we explore perturbation theory through the development of models to account for disturbing forces such as atmospheric drag, third body effects, and electromagnetic effects of the space environment. Although the addition of these forces make the trajectory equation impossible to solve analytically, their presence only adds a little algebra to update the numerical solutions developed in the first block of the course. Cadets are able to update their numerical computations to explore the effects of these perturbing forces on the satellite's path, comparing their intuition about these forces with graphs of resulting trajectories.

By the time we reach the third block, we've covered many of the essential elements of an introductory astrodynamics course with more advanced mathematical rigor than a traditional treatment. At this point in the course, our program bifurcates into two components. Individually, the students continue to further their learning about applied astrodynamics through the development and implementation of a self-selected project designed to give each cadet his or her own opportunity to explore a subset of these topics in depth. A more detailed explanation of the individual component of the course is described in the "Individual Research" section of this paper. Meanwhile, as a group, the class begins to stretch the brain's other hemisphere by considering the local and global shape of the universe, or at least the possibilities. Students learn important geometry and topology terminology and apply these ideas to various possibilities about the shape of space, such as a 3-torus (S2xS1), the 3D analog of a Klein bottle (K2xS1), tori that are glued with twists or reflections, and other 3D multi-connected manifolds. Additionally, we dissect the relativistic effects of gravitational forces on the local curvature of space-time. While these thought exercises of determining the topological properties of a space defined by a particular gluing are less computationally expensive than the lessons presented earlier in the semester, they are no less taxing on the brain, and they provide a nice change of focus for the end of the course.

Learning Community Model

This course attracts a variety of majors, and therefore each cadet brings a different set of strengths, weaknesses, interests, and background knowledge into our classroom. Rather than homogenize the class experience for all, a learning community model is adopted in an effort to allow each student to engage in the course as an individual contributor to the class. The in-class time is minimally spent on instructor lecture, and is mostly comprised of time allocated to working on problem sets similar to an inquiry-based learning model or doing hands-on learning activities. Below are descriptions of how the problems sets and the hands-on activities contribute to the formation of a learning community.

Problem Sets

Throughout the semester, there are six problem sets, referred to as group homework assignments. For the purpose of these assignments, the whole class is considered a "formal group," which at USMA means that the students are allowed to work together without having to officially document the details of their collaborations, thus allowing a free flow of information amongst the students. The assignment is broken into topics, and each of these topics is then tackled by a team of two or three students, with help as needed from other classmates.

On the day of the release of the assignment, the class holds a draft to determine which teams will work on each topic. Once a team has selected their topics, they know they will be responsible for solving all problems and answering all questions related to that topic, writing the formal paper submission of those problems, and presenting their work to the rest of the class such that each student gets to see the material. The team members are also given the responsibility of being the class SMEs (subject matter experts) on their topics, which means that, as other students require additional help on those topics, their classmates become the go-to source for help. From the date of the release of the assignment until the due date, much of the class time is spent working on these problems sets, with the instructor present merely as an additional resource and facilitator suggesting possible collaborations given the related nature of some of the topics.

The day the assignment is due, the entire class turns in one packet, signed by all of them. This packet contains the each set of solutions that were prepared by the teams of two or three cadets. In addition to the packet, each team presents their solutions. So often, in other courses, students are asked to present their solution to a problem that the rest of the class has already done, or at least attempted; this model means that students do not really have to worry about setting up the problem for an uninformed audience, and it can be difficult to keep the audience engaged during the rehashing of familiar material. In our class, since the material being presented is new to the audience, the audience is attentive and engaged. The presenters need to carefully consider the whole story they wish to present to the audience, including the background, the selection process for choosing a solution technique, the solution methodology, the final answer, and any warnings or potential pitfalls in working this sort of problem. The class is given the whole class period to present their material, and the classroom is reserved for several hours prior to the start of class so that cadets can prepare their notes on the boards during Dean's Hour (a non-class hour set aside for mid-day academic events such as exams, guest lectures, and other learning activities). The whole class is required to work together to determine a meaningful allocation of the presentation time, as some topics require more presentation time than others.

The grading of the assignment is broken into how the class performed as a whole (division of presentation time, preparation of the overall written packet), how the teams performed (accuracy of calculations, clarity of presentation content), and how the individual performed (presentation skills, confidence in the material, ability to respond to questions, ability to pose good questions to their peers).

Throughout the semester, the students begin to take ownership for various aspects of the course. They increase their ability to learn independently while leveraging the knowledge of their teammates. They develop confidence in their technical reading skills, as they pick through the course text, which is a graduate level textbook. They learn how to prepare presentations that are designed to teach their audiences and how to dynamically alter their presentations given the immediate feedback of confused stares or confident nods.

Hands-on Activities

In the words of Sophocles, "One must learn by doing the thing." Although a majority of the class time is devoted to working through and presenting the problem sets, whole-class activities are a great way to reinforce the material while pulling the class together to prevent divisions from forming along team-lines. These in-class hands-on activities are a way to make the complicated concepts of orbital mechanics and satellite dynamics tangibly accessible. A detailed description of each activity would be beyond the scope of this paper, however, a short list of activities is provided to give the reader a better understanding of what is meant by a hands-on activity: using string and sidewalk chalk to draw a scale model of the inner solar system plus Jupiter and the comet Encke; spinning in office chairs while matching foot-position to an elliptical path to ingrain how Kepler's Second Law is an application of the conservation of angular momentum; using hula hoops to represent orbits while learning the orbital elements and playing Orbital Element Simon Says; doing the relative motion dance by translating and rotating along increasingly complicated predefined paths while observing the relative motion of others in an effort to develop the equations for relative motion in a meaningful sense. In terms of community-building, there is an inherent bonding that occurs amongst those who go into battle and face fear together, and this past semester, the Space Math cadets all felt this as they turned, mortified, to see the Dean, a general officer, observing the whole class, each with a hula hoop around his/her waist during a game of Orbital Elements Simon Says.

Individual Research

This course attracts a variety of different majors: math, systems engineering, physics, operations research, and geography. It would be unrealistic to expect a diverse collection of students to come to this course with the same background knowledge or even the same reasons for taking the course, therefore it would be unfair to attempt to create a homogenous experience. After spending the first two thirds of the semester working through much of the foundational material, each student typically has a question in mind that they wish to explore further. At this point, each student submits an individual project preliminary proposal; they then work one-on-one with the instructor to refine their proposal into a tractable research problem.

Throughout the rest of the semester, each cadet works on his/her individual project, consulting with the instructor individually as needed. Due to the classroom environment of students using class time to work on the problem sets, these one-on-one meetings can take place with minimal interruption to individual schedules and do not detract from class-time focused on the current course material.

The final project is due as a formal written report and poster. The last week of class, the class holds a conference-style poster session, complete with invitations and snacks. As the guests roam the room, the cadets get an opportunity to present their posters to faculty guests from a variety of departments and other invitees such as coaches, friends, and Tactical Officers. The cadets are rightfully proud of their work, and it shows in the poster session as they explain their hard work like confident professionals. Examples of actual student projects have included: "What's going on at Yongbyon Nuclear Research Facility?" in which the cadet defined what type of coverage parameters would be required to call the coverage sufficient (range of passing altitudes, percentage of time covered, exclusion angles, etc.) and determined the parameters for a satellite constellation that would provide such coverage; "The Moon's Effect on a Satellite's Orbit" in which the cadet developed a numerical solution to the three body problem and plotted the results for a variety of initial conditions; "The Shape of Space: Cosmic Crystallography" in which the cadet worked through the current literature and developed numerical code that shows spikes that occur in histograms of pair-object distances in multi-connected spaces can be used to determine the topology of those spaces and was able to verify the results in the literature. There have also been a few projects relating to geo-based ISR missions, from monitoring Afghanistan to identifying the hottest nightlife in Rio, as well as number of projects on interplanetary travel including comparisons of using Lagrange points and riding the "Interplanetary Super Highway" versus spliced-conics and Hohmann transfers.

The feedback from the cadets regarding the poster session is highly positive. As a culminating experience, the poster session gives them an opportunity to reflect on all the work throughout the semester that brought them to this point, and the chance to share their work is rewarding. Additionally, the cadets get a chance to see the room filled with a variety of projects, indicating the wide range of applications for the material they have discovered through the course. So while the final project is an individual project, the poster presentation provides a means of sharing diverse ideas throughout the learning community.

Summary

The initial goal in developing this course was to provide interested cadets with a fundamental knowledge of astrodynamics and the underlying mathematics in an effort to promote Army Space Operations. Many cadets who have come through this course have left with dreams of becoming an astronaut or serving as a mission analyst as an FA40 officer. The students realize that there are opportunities to contribute to space missions within the Army, and that dreams of being an astrodynamicist do not require a NASA badge. The material in the course augments the material taught in the Space Physics course, and the cadets leave with a strong appreciation for the mathematics behind the physics in the design of satellite missions. The cadets' appreciation for the applicability of their lessons to military missions is evident with the high number of geocentric ISR missions proposed for the final project. The learning community model allows the class to cover the material both in breadth and depth, by having small teams explore topics deeply and then sharing their results amongst the class. Therefore, the "graduates" of Mathematics for Space Applications are well prepared to contribute to the Army Space community.

Lastly, our goal at West Point is to develop leaders of character. Each cadet in this course is charged with a tremendous responsibility, as each small team is not only responsibility for their own understanding of a given topic for each assignment, but also is charged with the responsibility of teaching their peers. As the cadets take ownership of the material and accept their roles as subject matter experts, their ability to lead their peers through the concepts is sharpened, their ability to read their audience and react respectfully is developed, and their confidence as learners and teachers is enhanced. Each of these traits contributes towards improving the development of these cadets in their path towards becoming leaders of character.

EI Videos

Michael Courtney, PhD and Lt. Col. (Ret.) Tom Slusher
United States Air Force Academy
Michael.Courtney@usafa.edu

Amy Courtney, PhD
BTG Research, PO Box 62541, Colorado Springs, CO 80962

Abstract

The Quantitative Reasoning Center (QRC) at USAFA has the institution's primary responsibility for offering after hours extra instruction (EI) in core technical disciplines (mathematics, chemistry, physics, and engineering mechanics). Demand has been tremendous, totaling over 3600 evening EI sessions in the Fall of 2010. Meeting this demand with only four (now five) full time faculty has been challenging. EI Videos have been produced to help serve cadets in need of well-modeled solutions to homework-type problems. These videos have been warmly received, being viewed over 14,000 times in Fall 2010 and probably contributing to a significant increase in the first attempt success rate on the Algebra Fundamental Skills Exam in Calculus 1. EI Video production is being extended to better support Calculus 2, Calculus 3, and Physics 1.

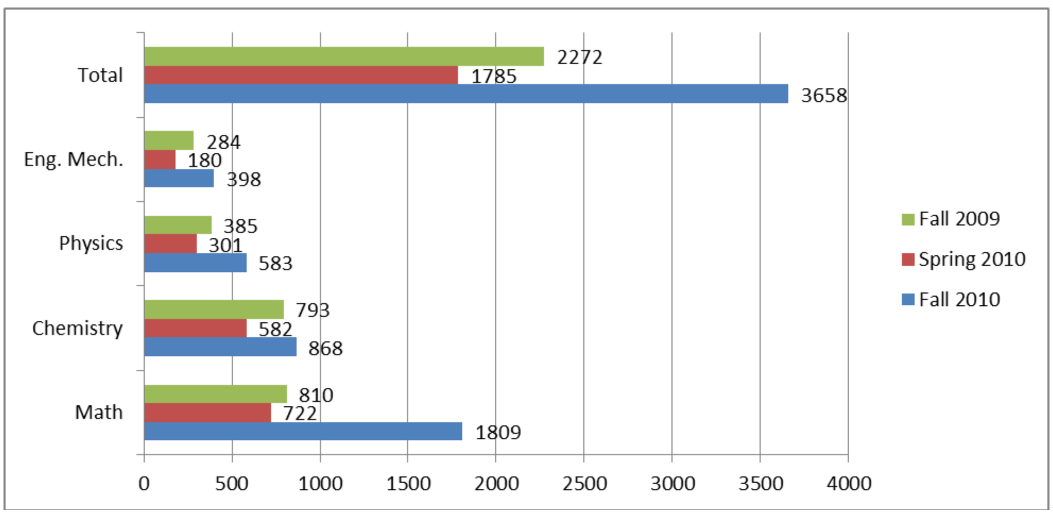

Figure 1: USAFA evening EI sessions (in person) by subject, Fall 2009-Fall 2010, EI Video support was added for every lesson in Fall 2010.

Introduction

Soon after USAFA began offering extra instruction (EI) services in core technical courses during evening hours (1800-2200), the demand and rapid growth revealed the need for force multipliers given the availability of only four (now five) full time faculty.[1] Figure 1 shows the number of evening EI sessions in each subject for the first three semesters of evening EI availability. In the Fall of 2010, demand for evening EI grew to 3600 evening EI sessions in the core classes, over 1800 sessions in the supported Math courses, including Pre-calculus, Calculus 1, Calculus 2, and Calculus 3, The Qualitative Reasoning Center (QRC) faculty quickly noticed that most EI needs fall into two phases: 1) modeling problem solving methods at cadet request and 2) coaching cadets as they work homework problems at the board. Because evening EI services are utilized disproportionately by cadets with weaker math backgrounds, it was observed that these cadets often need to see one or two worked examples (model phase) before the EI process can productively move to the coach phase. Modeling the same subset of assigned practice/homework problems repeatedly for different cadets quickly gave rise to the idea that a lot of the modeling phase could be accomplished by producing videos that can be posted on the USAFA intranet for cadets to view at their convenience, thereby leaving more EI face time for the coach phase of the EI process.

In Spring 2011, the USAFA QRC is working to build the available EI video library to at least one video pertaining to each lesson's assignment in Calculus 1, Calculus 2, Calculus 3, and Physics 1. We are in the process of prioritizing development for future semesters to serve the core Chemistry sequence and Physics 2. Manpower limits the ability to provide immediate support to a wider variety of problems or courses, since each 10 minute video requires between 1 and 2 hours to produce once the instructor is experienced in the process. Most instructors find themselves unsatisfied with their first few efforts and end up re-recording their first few videos.

EI Video Pedagogical Philosophy and Method

USAFA's academic departments do an excellent job presenting core technical material to cadets with average and above backgrounds in science and mathematics, and the institution is highly regarded for instructor availability for EI to meet the needs of cadets for whom the classroom presentation is not sufficient.[2] Evening EI and the EI Video programs are an extension of this availability. Figures 2 and 3 show that evening EI services are used disproportionately by groups traditionally underrepresented in the STEM disciplines and by cadets with below average academic backgrounds. (The academic composite is a measure of strength of a cadet's academic background and is intended to be a predictor of cadet GPA. The mean academic composite is between 3100 and 3200 for most incoming cadet classes.)

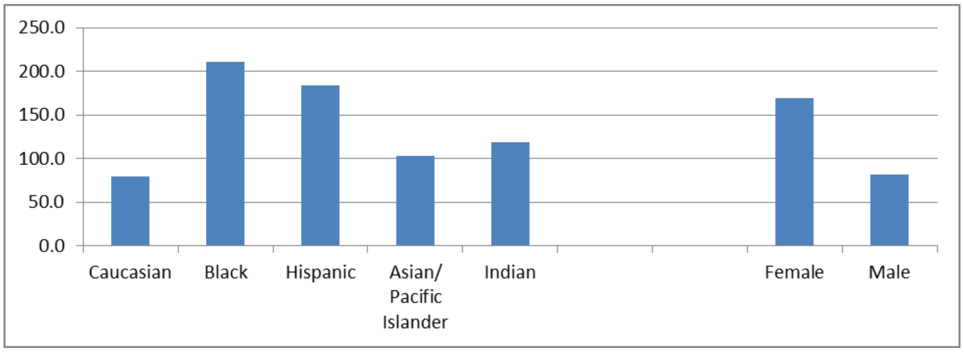

Figure 2: Evening EI usage as percent of representation at USAFA. For example, since female cadets represent 35.3% of evening EI visits and 20.9% of underclassmen, the number reported above is 35.3%/20.9% (multiplied by 100%) = 169%. Thus numbers above 100 represent disproportionately high usage by the group. Numbers below 100 represent disproportionately low usage by the group.

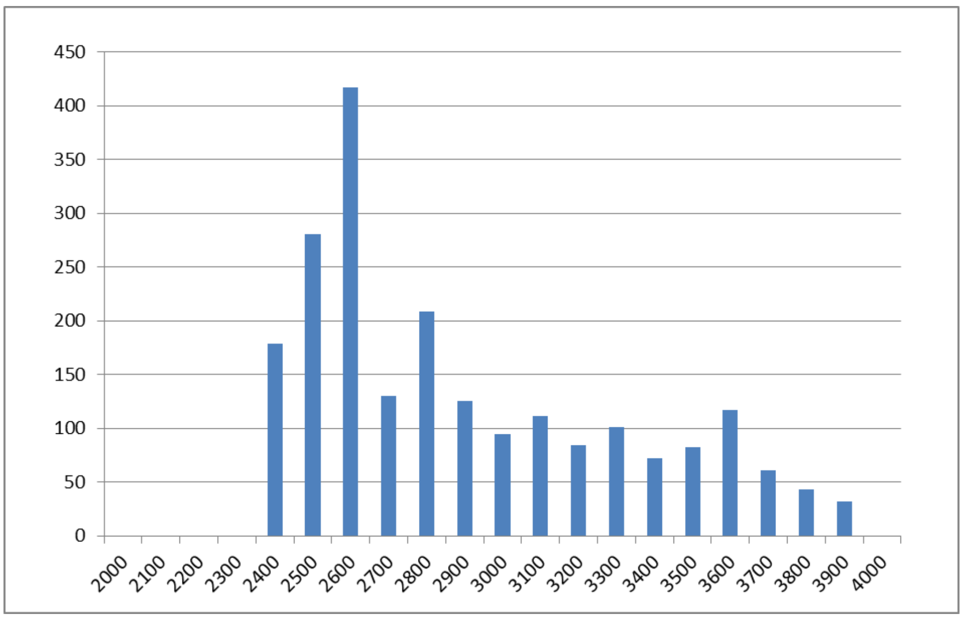

Figure 3: Evening EI usage as percent of representation at USAFA for academic composites 2000-4000. For example, since cadets with academic composites from 3800 to 3899 represent 4.00% of underclassman but only 1.75% of evening EI visits, the number reported above is 1.75%/4.00% (multiplied by 100%) =43.7%. Thus numbers above 100 represent disproportionately high usage by the group. Numbers below 100 represent disproportionately low usage by the group.

Recognizing that EI services are disproportionately utilized by students with weaker academic backgrounds, the pedagogy of the EI videos is aimed at the lowest quartile of students. This is a bit different from typical classroom instruction, which often tends to be aimed more at the middle. An example that might take only 5 minutes to discuss in class is slowed down to 10 minutes, and the video instructor is more intentional in most videos about developing a plan and working intermediate steps in greater detail.

Experience has shown that cadets who understand the typical classroom level of presentation are not the most likely to be seeking EI. Furthermore, it is easy enough for stronger students to fast forward material that is overly detailed, but cadets who need more detail simply cannot slow things down enough to see things that are not on the video, and they can't raise their hands and ask questions if the instructor on the video skips a step they need to see.

Inspiration for the EI videos was drawn from many sources, but with particular emphasis on the MIT Video Lecture portion of their open course ware (http://ocw.mit.edu) and to the MathTV channel at YouTube (http://www.youtube.com/user/MathTV). Simply directing cadet traffic to these and other available high-quality internet video resources was considered. However, in the time squeeze that is cadet life, cadets were not interested in a search for the video that demonstrates the topic at hand in just the right way every time they have a question. In contrast, when course directors provide links in the syllabus directly to close analogues to assigned homework problems, cadets are quick to access the available resource since they are sure it directly pertains to the task.

Additionally, instructors both teach what we know and impart who we are; therefore, we also preferred an approach to EI Videos that would promote officer development as well as academic success. Since the goal is to develop character and not just teach academics, most EI Videos include a brief introductory vignette, a 30-120 second segment before the pedagogical portion encouraging cadets to form better habits, pointing out the military or practical applications of the topics being discussed, and/or sharing personal experiences related to the topic or the training process. One of the cadets' favorite vignettes is an instructor attempting dance moves from “Saturday Night Fever” and then admitting, “When I try to dance, I look like a dufus – because I haven't practiced.” The vignette closes and transitions into the example problem with the admonition that without practicing the homework problems, watching EI Videos won't make them any better at math than watching “Dancing with the Stars” will make them a better dancer. Other vignettes feature an instructor with a barbell encouraging, “The math class is the weight room for the mind . . .” warning against “Five frequently fatal freshmen physics fantasies” [3] or holding a precision rifle and explaining the importance of mathematics in the profession of arms which is about “putting projectiles on target.” Some example videos have been uploaded to YouTube, because a lot about the vignettes is hard to explain in writing, but easy to perceive. [4][5][6]

Detailed production tips are described in the appendix. None of the video instructors have been terribly excited about how they look and sound on video. The camera adds 20 pounds and seems to magnify every wrinkle and mannerism, every “um”, “er”, and pregnant pause while one collects a thought and considers the next phrase. Confidence and improved ability come with practice. We have learned to get over our vanities and get the job done putting well considered solutions on video. Teaching on video is a great tool for breaking bad habits and smoothing one's presentation. One video instructor lost 30 pounds to better present a good example of lifelong fitness on camera and in the classroom. The path to growth is jumping in and trying it.

Results

The EI Videos have been very well received by cadets, administration, and faculty alike. In Fall 2010, EI Videos were viewed over 14,000 times, which compares favorably with the 1800+ in person EI visits in Mathematics. It seems self-evident that students who avail themselves of EI opportunities will perform better. Our analysis of Fall 2009 data showed Calculus 1 students who visited in person for evening EI prior to the Algebra Fundamental Skills Exam had a first attempt score average (76%) nearly as high as the course wide average (78%), even though they scored much lower (52%) than the course wide average (60%) on the Algebra portion of the placement exam.

Since we have no way to track which cadets are viewing EI Videos, a direct comparison with in person EI is not possible. However, the course wide first attempt pass rate of the Algebra Fundamental Skills Exam in Math 141 increased from 53% in Fall 2009 (before EI videos) to 63% in Fall 2010 (after EI videos).[7][1] This increase could be due to other factors, such as significant changes and an emphasis in practicing for this event in USAFA's First Year Experience (FYE) course. However, many cadets directly attribute their success on the FSE to the availability of EI videos, and they overflow with gratitude and appreciation when they meet the video instructor in person.

One might wonder whether the availability of video EI is robbing cadets of the character development and obviously superior instruction of face time with a real instructor. The data suggest otherwise. In Fall 2010, only Mathematics courses were supported with EI videos, and growth of in-person evening EI in Mathematics far outpaced other disciplines. The Department of Mathematical Sciences does not track in-person daytime EI for the entire Calculus 1 course. However, the course director and instructors have offered anecdotes (and some sign-in logs) suggesting that daytime EI for the course was also increased significantly in Fall 2010 compared with Fall 2009. So it seems that EI videos have the potential to both meet demand for the modeling aspects of EI and create increased demand for the coaching aspects. The demographics of the incoming cadets in 2009 and 2010 were relatively constant (within 2%) with respect to incoming standardized test scores and the percentage of groups that are commonly underrepresented in STEM majors, so the growth of in-person EI is not attributable to a dramatic change in the incoming class. Video EI is also useful for students who miss class due to illness, travelling, or intercollegiate athletics.

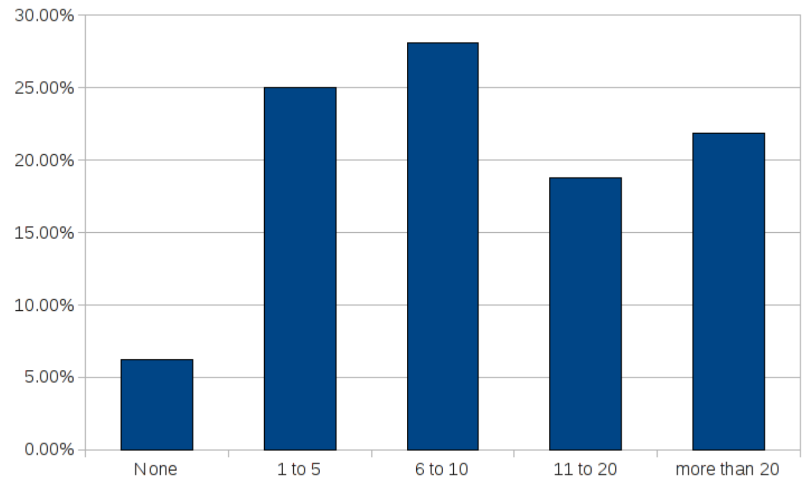

Figure 4: Results of an anonymous survey of 32 cadets who attended an evening EI session late in the semester regarding how many EI Videos they viewed that semester. About 2/3 of video views are spread evenly through the semester with the remaining 1/3 of video views in the 48 hours before graded reviews and final exams.

Figure 4 shows the results of an anonymous survey administered to students who attended EI one evening between the end of classes and the beginning of final exams. Only 2/32 (6%) of cadets had not watched any EI videos, showing that the videos were widely utilized among cadets attending evening EI. However, even though over 50 videos are available in the courses served, most (19/32) cadets report watching 10 or less, demonstrating that the cadets are utilizing videos as needed and not as a way to short circuit the homework process by going straight to every available video. Only 22% of respondents report watching more than 20 videos. These results agree with the total of 852 cadets in Calculus 1 viewing the videos available for that course approximately 8500 times. From a manpower point of view, providing this many in-person EI sessions would be a daunting task.

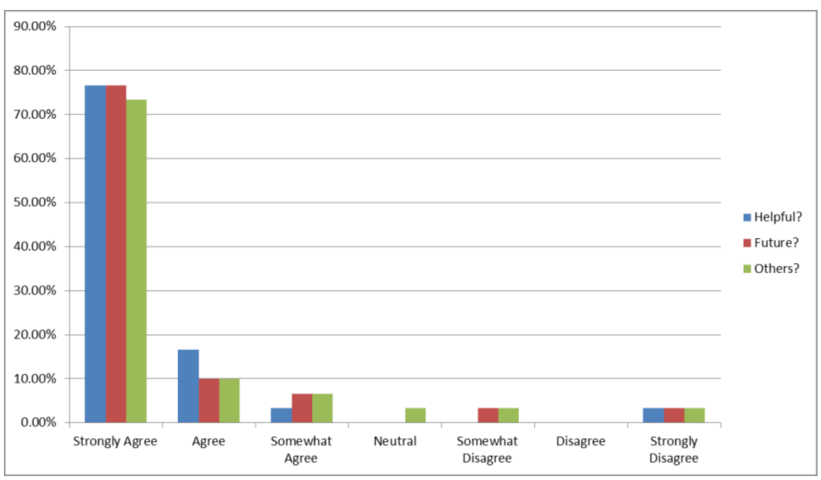

Figure 5: I found the EI videos helpful. (blue) I will watch EI videos in the future. (red) I will recommend EI videos to others. (green)

Figure 5 shows the survey results of the 30 students who reported watching EI Videos. Over 70% strongly agreed that the EI videos were helpful, planned to watch EI videos in the future, and would recommend EI videos to other cadets. Less than 10% reported any negative sentiments.

Conclusion

Cadets and instructors also report that the demonstration of technology in EI videos also greatly increases cadet comfort with these tools and encourages their use by cadets earlier in the course. Each math course has some graphing tools that are allowed and encouraged, and often weaker students put off learning them until late in the semester. Calculus 3 is particularly challenging for cadets who attempt to pass without learning either MATLAB or Mathematica, because these tools empower both two and three dimensional graphics, and they automate repetitive calculations with speed and accuracy that are seldom matched with pencil and paper. EI videos demonstrating the speed, accuracy, and power of these tools in working multiple integrals, executing vector operations, computing arc length, graphing vector functions, and solving optimization problems via LaGrange multipliers have proven both popular and effective in helping cadets grow into letting the technology handle the repetitive tasks and freeing them to concentrate on interpreting the problem, choosing the appropriate tools, and applying a sound problem solving methodology without getting bogged down in the computational minutiae. When leveraging technology in EI videos, we prefer to open with a vignette, proceed to identify the principles and develop the picture and plan on a white board, and only move to the computer at the execution stage of the solution, being deliberate to move back to the board or at least use a combination of verbal and graphical assessment before concluding.

Textbook updates and other curricular changes will probably render the first round of videos obsolete within a few years. Given the other responsibilities of QRC faculty, keeping the videos current with the curriculum will prove challenging, and the STEM departments typically do not have the available resources to assist with video production. One idea for the future is to have cadets themselves produce EI Videos as a course project or other assignment. We’ve also found that the high demand for videos can often produce slow downloading times on the USAFA intranet before graded reviews and final exams, and that the SharePoint occasionally becomes unavailable at inopportune times. We’ve begun hosting videos for some of the courses on outside servers and will be moving to redundant web hosting of all the videos in the near future both to ease the bottleneck on the single internal SharePoint server and to have alternate hosts when one is unavailable.

Appendix: Production tips

Preparation:

1. Work the problem in advance and plan a solution that is well presented including a vignette, big idea, picture, plan, evaluation, numerical solution, and assessment.
   Components:
   Vignette (optional): A 30-120 second skit or monologue describing the relevance of the topic to the military, encouraging good practice or study habits, or generally describing the importance of Math for an officer.
   Main Idea: Tell them what the problem is and what the big idea will be used to solve it.
   Picture: If at all possible, draw a picture that represents the problem.
   Plan: A plan with bullet points or numbers is a great outline for the solution you are about to present.
   Evaluation: This is the solving step where the plan is executed. Don't skip steps and remind the viewer of the justification for each step.
   Solution: Substitute in the numbers, describing the solution, including units, etc.
   Assessment: Explain how you know whether the answer makes sense in the context of the original problem. Are the units, magnitude, and sign what you expected? Why? Can the problem be bounded with simple estimates?
2. Plan how you will use board space to minimize erasing and poor presentation.
3. Shoot in a well-lit room and turn all lights on.
4. Plan to shoot in one take. It is easier to edit out mistakes from one take than to splice multiple short videos into a longer one, so just back up and start again from any mistake you might make.
5. Dress nicely. It will help your confidence. Some trial and error is often necessary in finding clothes that look good on camera. One instructor always wears a suit coat because his shirts never look good in back.
6. Position an academy banner or academy sports poster to the side of the board viewing area behind where you'll stand when just talking. Subliminal message: Being at a service academy is a tremendous privilege.
7. Look through the camera and define the viewing area with some horizontal and vertical lines so that you don't write out of the viewing area while presenting your solution. The viewing area should be completely above your waist, extend to the top of the white board or as high as you can reach and be 4-6 feet wide. Clean the board before beginning.
8. If you wish to include a vignette, plan it ahead of time and practice it. Keep it between 30 and 120 seconds.
9. Shoot for a video under 10 minutes after editing for Calculus 1 and 2; Calculus 3 and Physics videos often run longer.
10. Review some existing videos to get the gist of this.

Presentation:

1. Smile. Math is fun. Look happy. Be happy. Introduce yourself with a smile, transition between segments with a smile, and learn to smile while you are pausing and thinking. Smile when you make a mistake and when you re-enter after a mistake or pause. Cheesy is better than sour or angry.
2. Don't stop the camera. Splicing is much harder than editing out long pauses.
3. Be sure to present all the parts of your plan: vignette, main idea, picture, plan, evaluation, solution, and assessment.
4. Stand in front of the flag/poster when you're just talking, so you don't block the solution.
5. Don't obstruct the solution when you are pointing. Hold your open hand, facing forward, BELOW the part of the solution you wish to highlight.
6. Write neatly, level, and large enough to be legible on the video, but not so large you need to erase a lot to remain in the viewing area.
7. Eye contact with the viewer means looking at the camera. Instructors can turn the screen toward themselves, so checking the screen is perceived as eye contact.
8. Don't be afraid to try again. Most folks don't produce a video they are proud of on their first try. Copy the video to your computer, review it carefully, be intentional about the things you want to do differently, and try again on a different day.

Editing:

1. You need to copy the video to your hard disk for editing. Connect the camera to your computer via USB and look for the most recent file in the directory structure.
2. For many formats, editing is easily done with Avidemux, which is freely available and can be downloaded from a number of sources. Do a Google search and be sure you are downloading version 2.5 or later. Avidemux runs on Windows and Linux. The AVS video editor (for Windows) has more features and enables titles, scrolling credits, voice overs, and better support if including different media types. OpenShot is a free video editor for Linux with an impressive list of features, but it was extremely buggy and crash prone in our testing.
3. Avidemux is intuitive to use for cutting out unwanted material, mistakes, transitions, sneezes, etc.
4. Most single problem videos should be no longer than 10 minutes after editing.
5. Video, Audio, and Format specifications need to be carefully entered before you save the video, so that its formatting and compression are compatible with the other videos and preserve quality without being too big.
6. The specifications we use are: Video spec should be MPEG4-AVC. Video Encoding Mode should be "Video size, two pass." Target size should be 4 megabytes times the number of minutes: 10 minutes -> 40 megabytes. Other video encoding options should be left as their defaults.
7. Audio spec should be AAC (Faac). Other audio setting should be left as their

default.
8. Format should be mp4.
9. These specs are a good compromise between size and quality, enabling the written solution to be readable.
10. It takes most computers of recent vintage 2-6 times the video run time to fully compress the video.
11. Play the video yourself to ensure all went well before uploading.

Technical note: We like the JVC Everio GZ-MG335HU video camera ("normal definition") better than the JVC Everio GZ-HD320BU video camera ("high definition"). The normal definition camera saves as JPEG files where each frame is preserved without any loss. The high definition camera compresses the video in real time and the compression algorithm does not seem to be optimized for material written on the board. The normal definition camera allows compression to be controlled at the editing phase.

Acknowledgements
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While the missions of the mathematics departments at the service academies are quite similar, each has a different means of accomplishing its goals. By sharing information, we will be able to improve our programs by learning from each other. Hopefully, through Mathematica Militaris, these programs can continue to develop a common identity, gain recognition, and build an effective communication link.

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