CORE MATHEMATICS

DEPARTMENT OF MATHEMATICAL SCIENCES

U S M A

ACADEMIC YEAR 2013 – 2014

CORE MATHEMATICS AT USMA
AY 13-14

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CORE MATHEMATICS AT USMA
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INTRODUCTION

This document is designed to inform USMA faculty in the Math/Science/Engineering (MSE) Departments, and interested others, about the core mathematics program. Inside you will learn what mathematical skills and concepts you can expect from your cadets as they progress from admission to the end of the core math sequence. You will see some of the philosophy and growth goals that we have adopted in order to develop a diverse group of high school graduates into college juniors who are prepared to succeed in an engineering stem or in higher-level disciplinary study. You will read about our new interdisciplinary goal and Liaison Professor programs. These programs aim to achieve a more integrated experience for cadets by promoting coordination and collaboration between the Department of Mathematical Sciences and other academic departments. You will also learn the details of our program for identifying and reinforcing required mathematical skills for entering cadets.

We have included a detailed summary of course objectives for each core math course for this academic year. As part of our educational philosophy, we recognize that mastering conceptual knowledge is a difficult process requiring periodic review, practice, and consolidation. Therefore, we recommend that courses which rely heavily on portions of this conceptual material identify those portions to the cadet at the beginning of the course, and then reinforce student understanding as appropriate.

Also of special interest is the list of Mathematical Recall Knowledge. This is a modest list of basic facts that the MSE Committee has judged should be memorized by each cadet. Within the core math program, cadets periodically test their proficiency on this current and accumulated recall knowledge. For MSE courses that rely heavily on some subset of these recall skills, we again recommend that you identify these to your cadets at the beginning of the course, and then reinforce (and test) them as appropriate.

Applied mathematics is the process of appropriately transforming one form into a more useful form in order to reveal additional insight. This can be as simple as transforming an array of numbers into a sum; a function into its derivative; or a computer network into a bipartite graph. A historian who can transform the data of secondary sources into information that reveals the interactions of an earlier society is an applied mathematician in disguise! It is essential that we provide our cadets an experience where they can appreciate and apply the power of transformations to solve real problems.

Technologies are facilitating new transformations and have made other transformations obsolete. Hence, the teaching of mathematics at the undergraduate level is changing! While new technologies allow us to do innovative things today, experience strongly suggests that much of it will be only partly relevant, sometimes misleading, and occasionally wrong. We know that what we consider to be the fundamentals of mathematical knowledge change more slowly. Our role is to continually find the appropriate balance between technology and the fundamentals as mathematical pedagogy and content evolve. This year we have identified a list of fundamental concepts both non-technology and technology related for each of our core courses.

We hope you find the information in this booklet useful. We welcome your feedback on how we can better coordinate our programs, and on what information we can include here in order to help you succeed as a teacher who uses the tools from core mathematics. The Department of Mathematical Sciences updates this document annually in coordination with the MSE Committee. Please direct any comments to the former for inclusion in the next edition.

MICHAEL D. PHILLIPS
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Head, Department of Mathematical Sciences

EDUCATIONAL PHILOSOPHY

The Role of Core Mathematics Education at USMA

The mind is not a vessel to be filled but a flame to be kindled. -- Plutarch

Core mathematics education at USMA includes both acquiring a body of knowledge and developing thought processes judged fundamental to a cadet's understanding of basic ideas in mathematics, science, and engineering. Equally important, this educational process in mathematics affords opportunities for cadets to progress in their development as life-long learners who are able to formulate intelligent questions and research answers independently and interactively.

At the mechanical level, the core math program seeks to minimize memorization of a disjoint set of facts. The emphasis of the program is at the conceptual level, where the goal is for cadets to recognize relationships, similarities, and differences to help internalize the unifying framework of mathematical concepts. To enhance understanding of course objectives, major concepts are presented numerically, graphically, and symbolically. This helps cadets develop a visceral understanding that facilitates the use of these concepts in downstream science and engineering courses.

Concepts are applied to representative problems from science, engineering, and the social sciences. These applications develop cadet experience in modeling and provide immediate motivation for developing a sound mathematical foundation for future studies.

The core mathematics experience at USMA is not a terminal process wherein a requisite subset of mathematics knowledge is mastered. Rather, it is a vital step in an educational process that enables the cadet to acquire more sophisticated knowledge independently. Cadet development dictates that we must provide the cadet time for experimentation, discovery, and reflection. Within this setting, review, practice, reinforcement, and consolidation of mathematical skills and concepts are necessary and appropriate, both within the core math program and in later science and engineering courses.

Cadets completing the core math program will have developed a degree of proficiency in several modes of thought and habits of the mind. Cadets learn to reason deductively, inductively, algorithmically, by analogy, and with the ability to capture abstractions in models. The cadet who successfully completes the USMA core mathematics program will have a firm grasp of the fundamental thought processes underlying discrete & continuous processes, linear & nonlinear dynamics, and deterministic & stochastic processes. The cadet will possess a curious and experimental disposition, as well as the scholarship to formulate intelligent questions, to seek appropriate references, and to independently and interactively research answers. Most importantly, cadets will understand the role of applied mathematics – insight gained from transformations.

Adaptive Curriculum

Computing has changed profoundly-and permanently-the practice of mathematics at every level of use. College mathematics departments, however, often lag behind other sciences in adapting their curricula to computing, although considerable momentum is now building within the community for greater use in computing.…Computing can enhance undergraduate study in many ways. It provides natural motivation for many students, and helps link the study of mathematics to study in other fields. It offers a tool with which mathematics influences the modern world and a means of putting mathematical ideas into action. It alters the priorities of courses, rendering certain favorite topics obsolete and making others, formerly inaccessible, now feasible and necessary. Computers facilitate earlier introduction of more sophisticated models, thus making instruction both more interesting and more realistic. The penetration of computing into undergraduate mathematics is probably the only force with sufficient power to overcome the rigidity of standardized textbooks. The power of technology serves also an epistemological function by forcing mathematicians to ask anew what it means to know mathematics. Those who explore the impact of technology on education indict introductory mathematics courses for imparting to students mostly skills that machines can do more accurately and more efficiently.

The Department of Mathematical Sciences is committed to taking a lead in developing an effective new mathematics curriculum that attempts to foresee the mathematical needs of tomorrow's students. Our goal is to create a four-semester mathematics program that will enhance the mathematical maturity and problem solving skills of students.

Technology has reversed the roles of calculus and modeling. Traditionally, calculus determined the core program and modeling was used to support the application aspect of the program. Tomorrow, modeling and inquiry will determine the program with calculus and other mathematics subjects supporting the modeling portion of the program. Placing an emphasis on both discrete and continuous modeling broadens the role of mathematics to include transforming real world problems into mathematical constructs, performing analysis, and interpreting results. Placing an emphasis on inquiry provides opportunities for student growth in terms of learning how to learn, becoming an exploratory learner, and taking responsibility for one's own learning.

Our approach is to provide students with a broad appreciation and practice of mathematics through modern ideas and applications. The program is designed to provide student experiences through important historical problems and noteworthy contemporary issues as well. The goal is for students to establish a foundation from which to address unanticipated problems of the future. Examples of problem domains that will be considered include: the fair distribution of resources among nations; scheduling transportation resources; network design; financial models; population models; statistical inference; motion in space; optimization models; position, location, and geometric models; accumulation models; growth and change models; long-term behavior of systems; algorithm analysis, numerical techniques; linear and non-linear systems, and heuristic techniques. Some problem domains will be revisited several times at more sophisticated levels during the 4-semester program as students develop into more competent problem solvers.

The program will include contrasting approaches to problem solving such as: Continuous and Discrete; Linear and Non-linear; Deterministic and Stochastic; Deductive and Inductive; Exact and Approximate; Local and Global; Quantitative and Qualitative; Science (what is) and Engineering (what can be). For example, mathematics has the responsibility to develop a broad range of reasoning skills. Exposure to the art of reasoning will come through the process of induction and deduction. We will create learning opportunities where students move from the *puzzling* data to a suggested meaning (i.e., induction). Just as importantly, we will require students to move from the suggested meaning back to the data (i.e., deduction). This process of reasoning, along with other threads (e.g. approximation with error analysis, data analysis, discovery), will run throughout the core program.

Central to the entire program is the concept of problem solving in the modeling sense. This involves: 1) understanding the problem; 2) devising a plan to solve the problem; 3) carrying out the plan; 4) looking back to examine the solution in the context of the original problem.

SUPPORT OF USMA ACADEMIC GOALS

The Role of Core Mathematics in the USMA Education

USMA Academic Program Goal: The USMA academic program is designed to accomplish the overarching USMA Academic Program Goal.

> **USMA Academic Program Goal**
>
> Graduates integrate knowledge and skills from a variety of disciplines to anticipate and respond appropriately to opportunities and challenges in a changing world.

Academic Program Goals: There are 7 goals within the Intellectual Domain which support the above overarching USMA Academic Program Goal; of these, 4 are particularly pertinent to the Core Math Program:

- **Science, Technology, Engineering and Mathematics**
- **Lifelong Learning**
- **Communication**
- **Critical Thinking and Creativity**

Of these 4 goals, the Science, Technology, Engineering, and Mathematics goal places unique requirements on the USMA academic program which can be satisfied only by the Core Mathematics Program; the other 3 goals are addressed in a successive and progressive manner by both the Core Mathematics and other USMA programs. However, by virtue of its position at the beginning of the student's academic program and the large amount of consecutive contact time, the Core Mathematics Program has a large responsibility for initial growth in the latter 3 areas.

Science, Technology, Engineering, and Mathematics Academic Program Goal: The focus of the STEM Program Goal is on helping students to develop various modes of thought in a rigorous manner, to be able to utilize a formal problem solving process to solve complex and ill-defined problems in science and engineering, to develop scientific literacy sufficient to be able to understand and deal with the issues of society and our profession, and to effectively apply technology to enhance their problem solving abilities. The Math-Science Goal Committee deems students to have demonstrated evidence of achieving these goals when they can do the following:

What Graduates Can Do:

- ***Apply mathematics, science, and computing to model devices, systems, processes, or behaviors.***
- *Apply the scientific method.*
- ***Collect and analyze data in support of decision making.***
- *Apply an engineering design process to create effective and adaptable solutions.*
- *Understand and use information technology appropriately, adaptively, and securely.*

The two items in bold above are particularly pertinent to the Core Math Program.

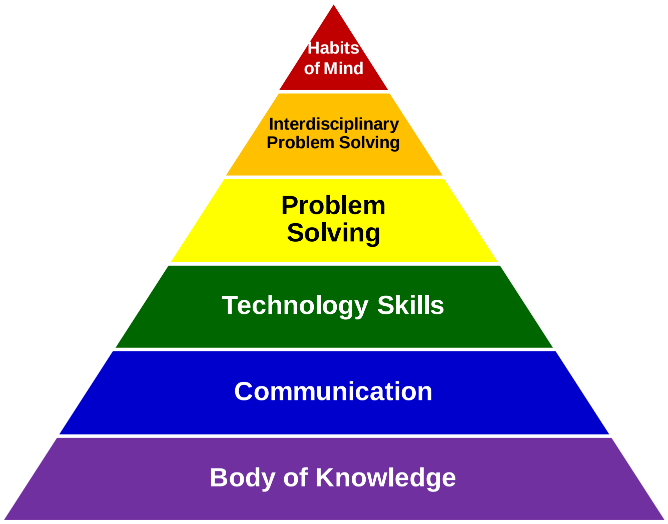
Lifelong Learning Academic Program Goal: The four semester contiguous core mathematics program plays an important role in transforming introductory college level students into independent learners prepared to begin study in their major discipline. The four-course connected core math curriculum provides a gradual transition from detailed to minimal direction. Each course provides the cadets with opportunities to take responsibility for their own learning. Embedded assessment indicators are used to track the role of the core mathematics curriculum in the continued intellectual development of our students.

Communication Academic Program Goal: Throughout the four-semester core mathematics curriculum, cadets are expected to communicate their problem solving process both orally and in the writing. They are introduced to the technical writing process and are evaluated according to the *substance*, *organization*, *style*, and *correctness* of their report. Each course also progressively develops cadets' ability to read and interpret technical material through increased reliance on the textbook and independent learning exercises, and to actively listen and participate in class instruction.

Critical Thinking and Creativity Academic Program Goal: Cadets completing the core mathematics program will have developed a degree of proficiency in several modes of thought and habits of mind. Cadets learn to solve complex problems through deductive and inductive reasoning, algorithmically, by analogy, and with the ability to capture abstractions in models. Interdisciplinary projects and activities are introduced throughout the core mathematics sequence developing our students' ability to transfer learning across disciplines. The critical thinking skills gained solving complex, ill-defined problems in the core mathematics program are a vital element in creating graduates that can think and act creatively.

Core Mathematics Program Goals

Core Mathematics Program Goals: The goals of the Core math program support the USMA Academic Program goals and the Math-Science goals. The six primary goals of the core math program are provided below.



- **Acquire a Body of Knowledge:** Acquiring a body of knowledge is the foundation of the core math program. This body of knowledge includes the fundamental skills requisite to entry at USMA as well as the incorporation of new skills fundamental to the understanding of calculus and statistics.

- **Communicate Effectively:** "Students learn mathematics only when they construct their own mathematical understanding." The successful problem solver must be able to clearly articulate their problem solving process to others.

- **Apply Technology:** Technology can change the way students learn. Along with increased visualization, computer power has opened up a new world of applications and solution techniques. Our students can solve meaningful real-world problems by leveraging computer power appropriately.

- **Build Competent and Confident Problem Solvers:** The ultimate goal of the core math program is the development of a competent and confident problem solver. Students need to apply mathematical reasoning and recognize relationships, similarities, and differences among mathematical concepts in order to solve problems.

- **Interdisciplinary Problem Solving:** In today's increasingly complex world, problems that leaders will face require the ability to consider a variety of perspectives. Mathematical analysis and results should not be accepted without understanding the social, economic, ethical and other concerns associated with the problem. The core math program seeks to expose cadets to problems with interdisciplinary scope. The goal is for cadets to consider what they have learned in other disciplines when faced with a problem requiring mathematical analysis. Additionally, cadets should appropriately apply mathematical concepts to support problems faced in other disciplines.

- **<u>Develop Habits of Mind:</u>** Learning is an inherently inefficient process. Learning how to teach oneself is a skill that requires maturity, discipline, and perseverance. The core math program seeks to improve each cadets reasoning power by introducing multiple modes of thought. These modes of thought include deduction, induction, algorithms, approximation, implications, and others.

Core Mathematics Goals and Outcomes
G1: Students display effective habits of mind in their intellectual process O1: Demonstrate curiosity toward learning new mathematics. O2: Reason and think critically through complex and challenging problems. O3: Demonstrate creativity and a willingness to take risks in their approach to solving new problems. O4: Display a sound work ethic, striving for accuracy and precision while maintaining strong resolve to complete problems in their entirety (i.e. persistence). O5: Think interdependently when working in groups. O6: Demonstrate the ability and motivation to learn new material without the help of the instructor. (Continued intellectual development).
G2: Students demonstrate problem solving skills O7: Transform problems into mathematical models that can be solved using quantitative techniques (use of the mathematics modeling triangle). O8: Select and apply appropriate mathematical methods as well as algorithmic and other computational techniques in the course of solving problems. O9: Interpret a mathematical solution to ensure that it makes sense in the context of the problem.
G3: When appropriate, students apply technology to solve mathematical problems O10: Use technology to visualize problems and, when appropriate estimate solutions. O11: Use technology to explore possible solutions to mathematical problems and by conducting a sensitivity analysis. O12: Confidently use existing and new technologies to leverage their skills.
G4: Students demonstrate an ability to communicate mathematics effectively O13: Write to strengthen their understanding of mathematics and integrate their ideas. O14: Discuss orally, their knowledge of mathematics to strengthen their understanding and integrate their ideas. O15: Understand and interpret written material (e.g, the textbook). O16: Listen actively to the instructor and student presentations.
G5: Students demonstrate knowledge of mathematics O17: Demonstrating competency of the fundamental skills required for entry into USMA. O18: Demonstrating competency of the fundamental skills required for each of the four core courses. O19: Demonstrating competency on major mathematical concepts for discrete math, linear algebra, probability and statistics, calculus and differential equations covered in the core program.
G6: Students demonstrate an ability to consider problems from the perspective of multiple disciplines including mathematics O20: Recognize the connections between disciplines when facing complex problems O21: Consider the implications of issues from other disciplines when formulating and utilizing mathematical models in problem solving O22: Include analytical approaches and mathematical models to help the problem solving process when faced with a problem from another discipline O23: Exposure in math courses to important problems that face the world around us

STUDENT GROWTH GOALS

Transforming High School Graduates into College Juniors

The greatest good you can do for [students] is not just to share your riches but to reveal to [them their] own. -- Benjamin Disraeli

When cadets enter the core math sequence, they are only a few months past their high school graduation. They represent a variety of backgrounds, levels of preparation, and attitudes toward problem solving. At the end of the core math sequence, these same cadets will have chosen an upper-division engineering sequence and a major (or field of study), will be halfway through the requirements for a BS degree, and will be aggressively and successfully tackling the more synthetic and open-ended challenges of their engineering science and engineering design courses. One of the responsibilities of the core math program is to move this diverse group of entering cadets from the former state to the latter. Our vehicle for doing this is the set of student growth goals outlined below. We feel that success in the MSE program at the Academy requires that each cadet mature in attitude toward the nature of problem solving and mathematics. Each cadet must grow confident in learning and applying mathematics, and each must develop facility in the skills and arts that allow them to apply mathematics and to collaborate with others. Careful coordination of the four core math courses allows a cadet to grow in each of the following areas. The result is a cadet who is prepared to perform the synthesis and to meet the open-ended challenges required by their chosen engineering stem.

BODY OF KNOWLEDGE:

- Incoming students are expected to be competent in algebra, trigonometry and pre-calculus upon entry to USMA. Although many students have been introduced to calculus prior to entering USMA, the curriculum assumes that students possess no prior knowledge of calculus.
- Students will develop competence in the following skills and topic areas through the core math program:

MA100

- Solve elementary algebra and geometry problems.
- Analyze, manipulate and, solve equations and inequalities.
- Recognize, sketch, and transform graphs and functions.
- Define functions and draw their graphs.
- Solve elementary inverse function problems
- Solve polynomial, and rational functions.
- Solve Exponential and logarithmic problems.
- Define trigonometric functions and solve problems involving trigonometric identities/equations.
- Formulate mathematical models using the various functions described in the course.

MA103

- Model and solve problems using discrete systems and difference equations.
- Apply matrix algebra to solve systems of equations.
- Model and solve problems using continuous equations.
- Use discrete and continuous models to make predictions.
- Understand the concepts of limits and rates of change, and apply these concepts to assist in understanding the long term behavior of mathematical models.
- Understand when it is advantageous to use technology, especially when developing models, exploring potential solutions, and iterating to find numerical solutions.

MA104

- Understand average and instantaneous rates of change.
- Understand the concept of the derivative and use this concept to model and solve problems.
- Be able to take <u>**basic**</u> derivatives by hand (polynomials, trig, exponential, and logarithmic functions)
- Understand vectors and vector functions
- Understand partial derivatives, directional derivatives, and gradients and use these concepts to model and solve problems.

MA205

- Understand accumulation in one and more variables (Single and Multiple Variable Integration)
- Be able to determine **basic** antiderivatives by hand (polynomials, trig, exponential and logarithmic functions)
- Use integrals to model and analyze simple physical problems.
- Evaluate integrals and iterated integrals.
- Be able to model real world problems using differential equations.
- Use analytical, numerical, and graphical techniques to solve 1^{st} and 2^{nd} order differential equations and systems of differential equations.

MA206

- Understand the basic concepts of probability, statistics, and random variables.
- Model applied problems using the fundamental probability distributions.
- Make inferences about a population using Point Estimation, Interval Estimation, and Hypothesis Testing.
- Use Linear Regression to make predictions.

TECHNOLOGY SKILLS:

- We expect incoming students to be comfortable using basic computer technology.
- Throughout the core curriculum, students will use both EXCEL and Mathematica to visualize, solve, analyze, and experiment with a myriad of mathematical functions (discrete, continuous, linear, non-linear, deterministic and stochastic).
- Following the core sequence, we expect students to be confident and aggressive problem-solvers that use technology to leverage their ability to solve complex problems.

COMMUNICATION

- Incoming students should be proficient in basic writing skills.
- Students are introduced to technical writing in their first core math course, and will progressively increase their technical writing abilities throughout the core program.
- Throughout the core curriculum, students will be asked to orally explain their mathematics through board briefings, in class discussions, and oral project presentations.
- Following the core math sequence, we expect students to adequately synthesize thoughts to explain their problem solving processes both orally and in a well written technical paper.

HABITS of the MIND:

- In studying mathematics, cadets learn good scholarly habits for progressive intellectual development.
- Cadets learn that mathematics is an individual responsibility that requires the motivation to learn, effort, time, and interaction with others.
- Cadets learn that mathematics requires an experimental disposition that in turn requires a curious mind, the ability to recognize patterns, the ability to conjecture, and the ability to reason by analogy.

PROBLEM SOLVING

- The core math program is a mathematical modeling and problem solving based curriculum.
- The core curriculum begins by reviewing essential skills in a modeling and problem solving context.

INTERDISCIPLINARY PERSPECTIVE

- The core math program routinely exposes students to problems from other disciplines requiring mathematical modeling and problem solving.
- Each core course includes projects and modeling experiences developed from interdisciplinary collaborations.

At the conclusion of the core math sequence it is anticipated that students are confident in their abilities to attempt to solve problems which they may or may not have ever seen before. In essence, we would like our students to confidently and aggressively pursue a solution process, when they are not really sure what to do.

Communications Goal

Level					
1	Students are expected to listen actively to a presentation by a student or instructor and begin to synthesize and emulate the same.	Students begin to read a mathematics textbook on their own and either synthesize the material or "know what they don't know" in order to frame an appropriate question.	Students receive guided instruction on the preparation of a technical report and begin to understand the process of articulating the problem solving process and its results. Students complete the course seeing what "right" looks like.	Students are introduced to the *Documentation of Written Work* and are expected to document all assignments according to the procedures outlined in the document.	Students begin to articulate the results of their work in board presentations.
2	Students are expected to listen actively to a presentation by a student or instructor and be able to synthesize and emulate the same.	Students continue to read a mathematics textbook on their own and either synthesize the material or "know what they don't know" in order to frame an appropriate question. Course projects generally require students to apply techniques to applications that they must read about on their own.	With minimal guidance regarding format, style and content, students prepare a technical report. The report should be essentially free of format/style errors (i.e. uses equation editor, appropriately includes figures/tables, appendices as needed). Articulates with reasonable clarity technical work from introduction/background of a problem to conclusion/recommendation. The report should be structured so that a reader can completely follow and understand the problem solving done, but is not expected to be either particularly cogent or elegant.	Students are expected to document all assignments according to the procedures outlined in the *Documentation of Written Work*.	Students continue to articulate the results of their work in board presentations. Project presentations are occasionally included in the course design.
3	Students are expected to listen actively to a presentation by a student or instructor and be able to synthesize and emulate the same.	Students continue to read a mathematics textbook on their own and either synthesize the material or "know what they don't know" in order to frame an appropriate question. Course projects generally require students to apply techniques to applications that they must read about on their own.	With little to no guidance regarding format, style and content, students prepare a technical report. The report should be essentially free of format/style errors (i.e. uses equation editor, appropriately includes correctly labeled figures/tables). Appendices are used only to include raw data, calculations, graphs, and other quantitative materials that were part of the research, but would be distracting to the report itself. The report refers to each appendix at the appropriate point (or points). Articulates with reasonable clarity technical work from introduction/background of a problem to conclusion/recommendation. The report should be structured so that a reader can completely follow and understand the problem solving done; it is expected to be reasonably organized and coherent.	Students are expected to document all assignments according to the procedures outlined in the *Documentation of Written Work*.	Students continue to articulate the results of their work in board presentations. Project presentations are occasionally included in the course design.
4	Students are expected to listen actively to a presentation by a student or instructor and be able to synthesize and emulate the same.	Students continue to read a mathematics textbook on their own and either synthesize the material or "know what they don't know" in order to frame an appropriate question. Students are assessed on their ability to self-teach themselves one section of the text book.	With no guidance regarding format, style and content, students prepare a technical report. The report should be free of format/style errors (i.e. uses equation editor, appropriately includes correctly labeled figures/tables). Appendices are used only to include raw data, calculations, graphs, and other quantitative materials that were part of the research, but would be distracting to the report itself. The report refers to each appendix at the appropriate point (or points). Articulates with reasonable clarity technical work from introduction/background of a problem to conclusion/recommendation. The presentation should be structured so that a reader can completely follow and understand the problem solving done; it is expected to contain appropriate substance, organization, style, and correctness.	Students are expected to document all assignments according to the procedures outlined in the *Documentation of Written Work*.	Students continue to articulate the results of their work in board presentations. Project presentations are occasionally included in the course design.

Technology Goal

Level			
1	Students are introduced to EXCEL and Mathematica and begin to use these programs to visualize, explore and solve mathematical programs.	EXCEL - Students are able to enter and plot data. Students use absolute references to model functions and refine models using the Solver. Students are able to iterate difference equations and systems of difference equations.	Mathematica – Students can enter a function and are introduced to the following commands: Plot, Solve, FindRoot, and Table. Students are able to perform basic Matrix operations on Mathematica.
2	Students continue to use EXCEL and Mathematica to visualize, explore and solve mathematical programs.	Students use EXCEL to assist in understanding the limit of a function.	Mathematica – Students increase their confidence using Mathematica by exploring problems involving continuous change. Students are able to take derivatives, evaluate them, and plot them. Students add the following commands to their repertoire: Limit, Norm, Cross, Dot, Plot3D, and ContourPlot. Students use the Help command to increase their ability to solve problems in Mathematica.
3	Students continue to use EXCEL and Mathematica to visualize, explore and solve mathematical programs. Students recognize the strengths and weaknesses and make choices regarding which technology to assist them.	EXCEL - Students use EXCEL to approximate integrals. Students are able to numerically approximate solutions to differential equations in EXCEL.	Mathematica – Students increase their confidence using Mathematica to explore problems involving accumulation and differential equations. Students are able to take definite and indefinite integrals. Students add the following commands to their repertoire: Integrate, NIntegrate, DEPlot, PhasePlot, and DSolve. Students increase their comfort detecting errors and fixing faulty code.
4	Students continue to use EXCEL and Mathematica to visualize, explore and solve mathematical programs.	Students use EXCEL to model empirical distributions to data sets and perform Monte Carlo Simulations	Mathematica – Students are able to use Mathematica to determine probabilities for any given probability distribution function (pdf or cdf). Students are able to determine critical points of the distribution functions from given probabilities using inverse techniques.

Habits of Mind Goal

Curiosity	Reasoning Critical Thinking	Creativity	Work Ethic	Thinking interdependently	Life Long Learning
Ask good questions	Identify relevant information Inventory needed information (known vs. unknown) Ask questions to clarify purpose or intent	Extend knowledge to new situations	Strive for accuracy & precision	Even when geographical separated	Learn continuously
Formulate questions to fill gaps between known and unknown	Make reasonable assumptions and recognize their effects Develop a plan to fill shortages	Analogy – retention of previous experiences to draw upon	Persist	Recognize potential contributions of each team member	Think about thinking – Develop processes Metacognition
Respond with wonderment and awe	Apply past knowledge Think flexibly - shift	Develop illustrations to clarify concepts	Attempt several methods without giving up.	Gather data from all sources	
	Apply induction –specific to general Apply deduction- general to specific	Establish connections between concepts	Remain focused on developing a solution strategy and implementing it.	Paraphrase another's ideas	
	Think first- manage impulsivity	Take responsible risks	Move beyond their comfort zone	Understand the diverse perspectives of others	
	Check & critique own work			Act responsibly in fulfilling group commitments	

MA100 – Precalculus
AY 13 – 14 Class of 2017
COURSE DESCRIPTION

MA100 prepares cadets with background deficiencies in algebra and trigonometry for the core mathematics program. The course develops fundamental skills in algebra, trigonometry, and functions, through an introduction to mathematical modeling and problem solving, providing the mathematical foundation for an introductory calculus course.

Note: Cadets taking MA100 will follow this course with MA101, Mathematical Modeling and Introduction to Calculus and then MA104, Calculus I. Together MA100 and MA101 count as MA103 in the 8TAP and cover all the course objectives and embedded skills in MA103.

The course is broken down into four blocks of instruction. The block objectives are as follows:

Block 1: Review of Fundamentals

Objectives:

(1) Understand the properties of real numbers (e.g. integer, rational, irrational), negative numbers, fractions, sets and intervals, and exponents and roots.

(2) Use basic properties of arithmetic operations to identify equivalent algebraic expressions and to re-express a given expression in various alternative forms to include
- Expanding or simplifying expressions
- Writing radical expressions as exponents and exponential expression as radicals
- Simplifying fractions and compound fractions.

(3) Explain what it means to "solve" an equation or inequality, and do so for various kinds of examples.

(4) Use a Cartesian coordinate system to plot and identify points in a plane and graph a function.

(5) Apply these skills in a wide variety of practical situations.

Block 2: Functions

Objectives:

(1) Understand the definition of a function and be able to evaluate a function at any value, numeric or symbolic.

(2) Be able to sketch the graph of a given function, to include piecewise functions, and be able to determine the function's domain and range.

(3) Given the graph of a function $y = f(x)$, know and understand the transformation that
- shifts it vertically and horizontally
- shrinks it vertically and horizontally
- reflects it about the x and y axis
- combines shifts, shrinks, and reflects.

(4) Understand the definitions of the following and be able to graph them:
- polynomial functions
- exponential functions
- logarithmic functions.

(5) Apply these skills in a wide variety of practical situations.

Block 3: Trigonometry

Objectives:

(1) Understand the unit circle and the trigonometry of right triangles.

(2) Understand the definition of and be able to graph trigonometric functions.

(3) Understand the periodic properties of trigonometric functions and the transformations that shift, stretch and reflect trigonometric functions.

(4) Understand the laws of sines and cosines.

(5) Apply these skills in a wide variety of practical situations.

Block 4: Matrix Algebra

Objectives:

(1) Understand and apply common vector and linear algebra concepts to include:

- vector addition and subtraction
- scalar multiplication with vectors and matrices
- matrix addition, subtraction, and multiplication
- matrix inverses
- Elementary row operations to reduce a matrix.

(2) Formulate a model to represent certain situations with a vector, a matrix, or a system of linear equations.

Examples of Embedded Skills in MA100 which Support Course Objectives

1. Know the definition and understand the properties of real numbers, negative numbers, fractions, sets and intervals, and absolute values.
2. Be able to write radical expressions using exponents and exponential expressions using radicals.
3. Be able to use special product formulas to find products of (expand) and factor algebraic expressions.
4. Be able to simplify, add, subtract, multiply and divide fractional expressions.
5. Know the definition of a linear equation, and be able to solve linear equations.
6. Given an equation, be able to solve for one variable in terms of others.
7. Be able to solve linear inequalities, nonlinear inequalities, inequalities involving a quotient, and inequalities involving absolute values.
8. Be able to sketch a graph by plotting points, and find the x and y intercepts of the graph of equations.
9. Understand the definition of function.
10. Be able to evaluate a function (to include piecewise functions) at any value, numeric or symbolic.
11. Be able to find the domain and range of a function.
12. Understand average rate of change, how it is calculated and be able to apply the concept to problem solving.
13. Given the graph of $y = f(x)$, know and understand the transformation that shifts it vertically and horizontally and be able to sketch it.
14. Be able to combine functions and form new ones through algebraic operations on functions.
15. Be able to determine the end behavior of polynomials.
16. Be able to define complex numbers.
17. Understand the definition of exponential and logarithmic functions.
18. Understand the definition of the natural logarithm and its properties and be able to evaluate them.
19. Given a situation for circular motion, be able to find linear and angular speed.
20. Understand the definition of trigonometric functions.
21. Be able to express one trigonometric function in terms of another.
22. Be able to “solve a triangle” by determining all three angles and the lengths of all three sides.
23. Know the Law of Sines and Cosines.
24. Be able to graph trigonometric functions and their transformations.
25. Be able to use the substitution, elimination, and graphical methods to solve both linear and nonlinear systems of equations.
26. Be able to determine the number of solutions of a linear system in two variables.
27. Understand the definition of vectors, what they represent, their graphical representation, and their proper mathematical notation.
28. Be able to find the horizontal and vertical components of vectors.
29. Compute the dot (scalar) product of two vectors.
30. Solve systems of linear equations graphically and by substitution.
31. Solve systems of linear equations in several variables.
32. Understand the 3 possible number of solutions that a system of equations can have. For a linear system of equations with 2 variables, understand what the solutions mean graphically.
33. Use elementary row operations to solve a linear system of equations.
34. Find sums, differences, and scalar products of matrices.
35. Be able to compute the product of matrices (perform matrix multiplication).
36. Write a linear system as a matrix equation.
37. Understand how the inverse of a matrix can be used in mathematical operations.

MA103 – Mathematical Modeling and Introduction to Calculus

AY 13-14 - Class of 2017
COURSE DESCRIPTION

MA103 is the first course of the mathematics core curriculum. It emphasizes applied mathematics through modeling and using effective problem solving strategies and modeling theory to solve complex and often, ill-defined problems. The course exercises mathematical concepts while nurturing creativity, critical thinking, and learning through activities performed in disciplinary, interdisciplinary, and multidisciplinary settings. Special emphasis is placed on introducing calculus using continuous and discrete mathematics through applied settings. The course exploits a variety of technological tools to develop numerical, graphical, and analytical solutions that enhance understanding. The course is broken down into three blocks of instruction, all focusing on connections to Calculus. The block objectives are as follows:

Block 1: Modeling with Discrete Dynamical Systems (Discrete Differential Equations)

Objectives:

(1) Transform situations involving change at discrete intervals into mathematical models.
(2) Solve a discrete dynamical system (DDS) numerically (through iteration), graphically, and analytically (for linear cases).
(3) Analyze the long-term behavior and the equilibrium of a DDS.
(4) Interpret the results of solving a DDS for a given problem.

Block 2: Modeling with Matrices

Objectives:

(1) Understand and apply common vector and linear algebra concepts to include:
- vector addition and subtraction
- scalar multiplication of a vector and matrix
- matrix multiplication
- determinants and inverses
- elementary row operations
- eigenvalues and eigenvectors.

(2) Formulate a model to represent certain situations with a system of linear equations.
(3) Solve systems of equations manually using linear algebra techniques and using technology.
(4) Create a model by formulating a system of recursion equations. Solve the model numerically (through iteration), graphically, and analytically (for linear cases).
(5) Analyze the equilibrium and the long-term behavior of a system of recursion equations using the concepts of limits, rates of change, and eigenvalues and eigenvectors.

Block 3: Modeling with Continuous Functions

Objectives:

(1) Distinguish the difference between discrete and continuous scenarios.
(2) Explain the concepts of average and instantaneous rate of change as applicable to the definition of the derivative.
(3) Understand the analytic, geometric, and physical interpretations of the derivative.
(4) Distinguish common families of functions (linear, exponential, power, and trigonometric), their properties, and how their parameters impact their shape.
(5) Select the appropriate function and formulate a model to accurately analyze a specified data set. Judge the limitations and capabilities of this model.
(6) Evaluate model results to answer questions or make predictions about the situation being investigated.

Examples of Embedded Skills in MA103 which Support Course Objectives

1. Given an initial condition, iterate a difference equation.
2. Determine equilibrium values for recursion equations algebraically, numerically, and graphically.
3. Determine the long term behavior of a discrete dynamical system using its analytic solution.
4. Determine an analytic solution for homogeneous and non-homogeneous linear Discrete Dynamical Systems.
5. Algebraically verify an analytic solution to an initial value problem.
6. Find the sum and the difference of two vectors.
7. Multiply vectors by a scalar / matrix / vector. Compute the dot product of two vectors.
8. Multiply a matrix by a scalar / matrix / vector.
9. Solve systems of equations (2x2) by hand using substitution, row reduction, and the inverse of a matrix.
10. Use technology to solve systems of equations of virtually any size using row reduction or the inverse matrix method.
11. Understand graphical interpretation of an eigenvector and eigenvalue and how to compute each for a 2 x 2 matrix.
12. Using eigenvalues, eigenvectors, and vector decomposition to develop closed form solutions to initial value problems.
13. Using the definition of a function, determine which relationships are functions and which are not.
14. Describe the four different representations of functions.
15. Determine the limit of a function using graphical and numerical means.
16. Determine if a function is continuous, both at a point and on an interval.
17. Compute average rates of change to include slopes of secant lines, and approximate instantaneous rates of change by exploring the limits of the slopes of secant lines as well as calculating derivatives.
18. Know how parameters affect the shape of linear, exponential, power, and trigonometric functions; using one of these functions, evaluate a model's utility and make comparisons by computing its sum of squared errors (SSE) and its coefficient of determination (r^2).
19. Understand the graphical, algebraic and physical interpretations of the derivative.
20. Know, understand and apply the definition of the derivative.
21. Apply the definition of the derivative to find derivatives of functions.
22. Use the concepts of rates of change and derivatives to characterize the stability of the equilibrium values of non-linear difference equations.

MA103 makes important contributions toward achieving the goals of the overall academic program. In addition to reinforcing fundamental concepts, introducing students to interdisciplinary problem-solving, facilitating the integration of technology into problem-solving processes, and introducing students to problem solving strategies to address ill-defined problems, MA103 establishes a foundation for many critical concepts that are reinforced and used throughout the core math program and beyond. The following table lists some of the math concepts introduced in MA103 and indicates where they connect with the core math program and beyond.

MA103 Lesson Title	Supported Courses and Related Applications
Math Modeling (2 Lessons)	Mathematical Models and Math Modeling Process (Calculus Textbook, Stewart 1.2)*
Sequences	Arithmetic and Geometric Sequences (Calculus Textbook, Stewart 11.1)*
Long Term Behavior and Limits (2 Lessons)	MA205 (Infinite series, Long term behavior of solutions of diff eqns) MA364 (Fourier series solutions to PDEs)
Analytic Solutions for Linear Discrete Differential Equations (3 Lessons)	MA205 (Modeling with differential equations, Analytic solutions to differential equations, Models of population growth)
Introduction to Vectors (2 Lessons)	MA104 (3D Coordinate System, vectors, dot products) MA205 (Vector integration, projectile motion) PH201 (Vectors, kinematics, free body diagrams, work, torque) MA364 (Vector calculus)
Matrix Algebra and Solving Systems of Linear Equations using Matrices (4 Lessons)	Intro to Matrices and Matrix Algebra (not covered elsewhere in the core program); Inverses, Row Reduction, Determinants, Identity Matrices. These topics are required material for the Fundamentals of Engineering Exam.
Systems of Recursion Equations (Probability)	MA206 (Markov Chains, Markov Processes, Probability Distributions for Discrete Random Variables)
Eigenvalues and Eigenvectors and Analytic Solutions to Systems of Recursion Equations (6 Lessons)	MA205 (Solving systems of differential equations) MA364 (Solving 2^{nd} order differential equations)
Properties of Functions	Types of Functions (Calculus Textbook, Stewart 1.2)* Transformation of Functions (Calculus Textbook, Stewart 1.3)*
Limits	MA104 (Limits) MA205 (Improper integrals)
Continuity	MA104 & MA205 (Continuity)
Average and Instantaneous Rates of Change	MA104 & PH201 (Velocity Problems)
Derivatives and Rates of Change (2 Lessons)	MA104 (Tangent and Velocity Problems, Derivatives and Rates of Change)
Linear Functions (Parameter Estimation, Modeling)	Math Models: Linear Functions (Calculus Textbook, Stewart 1.2)*
Introduction to Power Models	Math Models: Polynomials and Power Functions (Calculus Textbook, Stewart 1.2)*
Power Functions (Parameter Estimation; Modeling) (2 Lessons)	MA104 (Power Rules for Differentiation) Transformation of Functions (Calculus Textbook, Stewart 1.3)*
Introduction to Exponential Models & Exponential Functions (Parameter Estimation & Modeling) (2 Lessons)	MA205 (Newton's Law of Cooling, Exponential Growth Models) SS201 (Time Value of Money)
Introduction to Trig Functions & Parameter Estimation & Modeling (2 Lessons)	Math Models: Trig Functions (Calculus Textbook, Stewart 1.2)* MA104 (Derivatives of Trig Functions)
Model Evaluation (3 Lessons)	MA206 (The Analysis of Variance, The Linear Regression Model, Estimating Model Parameters)

*Denotes critical introductory calculus material currently not covered in MA104.

MA104 - CALCULUS I
AY 12-13 - Class of 2016
COURSE DESCRIPTION

MA104 is the second semester of the mathematics core curriculum capitalizing on MA103's introduction to vectors and calculus. This Differential Calculus course emphasizes the conceptual understanding of single and multivariable differentiation using modeling and problem solving techniques. Applications of the derivative such as optimization, rates of change in one and several variables, motion in space, and intersection/collisions are used to motivate the study of calculus. The course provides the basic mathematical foundation for further studies in mathematics, the physical sciences, the social sciences, and engineering.

The course is broken down into three blocks of instruction focusing on differential and vector calculus. The objectives for each block are as follows:

Block 1: Single Variable Differentiation

Objectives:

(1) Understand the relationship and differences between average rate of change and instantaneous rate of change within the context of graphic, numeric, and algebraic representations.
(2) Determine and evaluate the derivative of functions using the properties of the derivative as well as applying differentiation rules.
(3) Find the derivatives of implicitly defined functions.
(4) Model and solve problems using rates of change, as well as related rates of change.
(5) Determine the local minima/maxima of a function and justify them with an appropriate test.
(6) Determine the absolute minimum/maximum of a function using the Closed Interval Method.
(7) Model and solve single variable optimization problems.

Block 2: Vector Functions and Geometry of Space

Objectives:

(1) Develop parametric equations and understand how to use them in modeling problems.
(2) Understand the graphical and physical interpretation of vector functions and their derivatives.
(3) Calculate and interpret the dot product.
(4) Calculate and interpret the cross product.
(5) Develop and interpret mathematical models used to depict scenarios involving lines and planes in space.
(6) Differentiate vector functions and understand the corresponding graphical and physical interpretation of the resulting derivatives.
(7) Utilize vector functions to model and analyze the two-dimensional motion of an object or the three-dimensional relative proximity of two objects moving through space.

Block 3: Multivariable Differentaion

Objectives:

(1) Understand the graphical, numerical, and algebraic interpretations of functions of two or more variables.
(2) Calculate and interpret partial derivatives for a multivariable function.
(3) Calculate and interpret directional derivatives for a multivariable function.
(4) Calculate and interpret the gradient vector for a multivariable function.
(5) Determine the local minima/maxima of a function of two variables and justify them with an appropriate test.
(6) Determine the absolute minima/maxima of a function of two variables on a closed and bounded set.
(7) Model and solve problems involving the optimization of a function of two variables.
(8) Understand the geometry of a constrained problem and solve constrained multivariable optimization problems using the Method of Lagrange Multipliers.

Examples of Embedded Skills in MA104 which Support Course Objectives

1. Approximate instantaneous rates of change using average rates of change or, graphically, approximate the slope of the tangent line at a point on a given curve using the slopes of secant lines.
2. Understand what it means for a function to have a limit, and approximate the limit of a function using graphical and numerical means.
3. Understand the mathematical and geometric definitions of continuity as well as the three different types of discontinuities at a point, and apply this understanding to determine if a function is continuous, both at a point and on an interval.
4. Understand the relationship between average rate of change and instantaneous rate of change graphically, numerically, and algebraically.
5. Interpret the physical meaning of the rate of change in the context of a given application.
6. Know and understand the definition of the derivative and apply the definition to find derivatives of basic polynomial functions.
7. Find the derivative of polynomial, exponential, logarithmic, and trigonometric functions, utilizing the properties of the derivative (constant multiple, sum and difference rules), power rule, product rule, quotient rule, and chain rule.
8. With an understanding of implicitly versus explicitly defined functions, model problems involving quantities that change over time (i.e., "related rates" problems) and solve them using implicit differentiation.
9. Determine the critical number(s) of a differentiable single-variable function and classify them as local and absolute (or global) extreme values using an appropriate test (Closed Interval Method, First Derivative Test, Second Derivative Test, or a graphical depiction of the function).
10. Model and solve single-variable optimization problems.
11. Understand that parametric equations determine the coordinates of a point on a curve using functions of a common variable, and determine the Cartesian equation of a function from a set of parametric equations, and vice-versa.
11. With an understanding of the Cartesian coordinate system, determine the Euclidean distance between two points in a three-dimensional space. Understand how to project a point onto a plane.
12. Understand what a vector is and the properties of vectors, to include algebraic and graphical depictions of vector addition, subtraction, and scalar multiplication.
13. Understand what a unit vector is and how to calculate the unit vector for any given vector.
14. Understand how vectors can be used to describe several forces acting on an object, and how the resultant force is the sum of these vectors.
15. Determine a vector between two points.
16. Understand the definition of the dot product and how to compute it by hand and with a computer algebra system.
17. Use the dot product to (a) find the angle between two vectors, (b) determine if two vectors are orthogonal, and/or (c) find the work done by a force.
18. Understand what vector and scalar projections are and how they are computed.
19. Understand the definition of a cross product and how to compute it by hand and with a computer algebra system.
20. Use the cross product to (a) find a vector that is orthogonal to two given vectors and (b) model and solve problems involving torque.

21. Be able to express the following in three-dimensional space: (a) a line using a point and a vector; (b) a line in space using a vector equation and a set of parametric equations; (c) linear equation of a plane in space using its normal vector; (d) an equation of a plane in space given either a point on the plane and a vector normal to the plane, or three points on the plane that do not lie on the same line; (e) the distance from a point to a given plane; (f) the distance from a point to a line
22. Understand the definition of a vector function and be able to find the domain of a vector function.
23. Understand how to calculate rates of change for vector functions and understand the physical interpretation of the derivative of a vector function, to include the relationship between the position, velocity, and acceleration vectors for an object's motion in space.
24. Be able to use the parametric equations of motion in order to solve projectile motion problems, to include (a) finding the velocity and acceleration vectors when given the position vector and an initial condition; (b) determining if the paths of two objects intersect given parametric equations modeling the paths of the objects in space; and (c) determining whether two objects collide, given parametric equations modeling the paths of the objects in space.
27. Given a function of two variables, determine the domain of the function.
28. Develop a general understanding of the use and meaning of level curves and level surfaces to represent functions of two and three variables, respectively.
29. Understand the definition of a partial derivative and describe geometrically what a partial derivative is for a function of two variables.
30. Approximate the partial derivative of a function of two variables at a point using average rates of change.
31. Given a function of several variables, calculate the partial derivatives.
32. Understand what a directional derivative is in terms of a rate of change and, given a function f of two variables, find the directional derivative of f at a given point in any direction.
33. Understand that the gradient vector gives the direction of the greatest increase in functional value at a given point for a differentiable function and, given a function of two variables, find the gradient vector at a specified point.
34. Find the vector that is normal to the level curve $g(x, y) = k$ at a specified point.
35. Understand that the magnitude of the gradient vector gives the maximum rate of change of differentiable function at a given point and how, geometrically, the gradient vector is related to level curves.
36. Given the surface $z = f(x, y)$, defined by a function which has continuous partial derivatives over some region $\boldsymbol{R}$, examine the level curves for possible maximum or minimum values.
37. Determine the critical point(s) of a differentiable multivariate function of two variables, $f(x,y)$, and classify them as local and absolute (or global) extreme values using an appropriate test (Closed Interval Method, Second Derivatives Test, or a graphical depiction of the surface).
38. Model and solve constrained multivariable optimization problems for functions of more than one variable.
39. Solve optimization problems for functions of several variables subject to a single constraint using the Method of Lagrange Multipliers.

MA205 - CALCULUS II
AY 13-14 – Class of 2016
COURSE DESCRIPTION

MA205 is the third semester of the mathematics core curriculum. This course provides a foundation for the continued study of mathematics and for the subsequent study of the physical sciences, the social sciences, and engineering. MA205 covers single and multivariable integral calculus and ordinary differential equations. Mathematical models motivate the study of topics such as accumulation, differential equations, motion in space, and other topics from the natural sciences, the social sciences, and the decision sciences. The course is divided into four blocks of material:

Block 1: Single Variable Integration and Motion in Space

Objectives:

(1) Evaluate net change using approximating rectangles to estimate area under a curve.
(2) Use integrals to model and analyze simple physical problems.
(3) Evaluate integrals and iterated integrals.
(4) Parameterize curves and analyze parameterized functions.
(5) Apply single variable integral calculus to vector functions to model and analyze motion problems.

Block 2: Modeling with Ordinary Differential Equations

Objectives:

(1) Model problems involving growth/decay, motion, heating/cooling, and mixing.
(2) Approximate a solution to a first order differential equation with graphical (slope fields) and numerical (Euler's method) techniques.
(3) Solve first and second order differential equations analytically (1^{st} order: separation of variables, integrating factor; 2^{nd} order: characteristic equation).

Block 3: Series Solutions to Differential Equations and Systems of Differential Equations

Objectives:

(1) Determine the power series solution to a differential equation by substituting the applicable forms of the power series into the differential equation and calculating the coefficients of the power series solution.
(2) Model multivariable problems as systems of first order ordinary differential equations and solve analytically using eigenvalues and eigenvectors.
(3) Determine and describe the long term behavior of a system of differential equations using analytical, graphical, and numerical methods.

Block 4: Multivariable Integration and Applications

Objectives:

(1) Use a double Riemann sum to estimate volume under a surface.
(2) Evaluate an iterated integral over a rectangular region
(3) Find the volume of any solid whose base is a specified region in the *xy* plane and bounded above by a given function $f(x, y)$.
(4) Use an iterated integral to compute volume, population, and other values that can be accumulated in two dimensions.
(5) Be able to convert points and equations between Cartesian and polar forms to evaluate double integral**s**

Examples of Embedded Skills in MA205 which Support Course Objectives

1. Approximate net change in two and three dimensions using Riemann sums.
2. Find the anti-derivative of basic polynomial, exponential, logarithmic, trigonometric, rational, and power functions.
3. Understand the Fundamental Theorem of Calculus and how it establishes the relationship between the derivative and the antiderivative.
4. Use the Fundamental Theorem of Calculus to evaluate definite integrals of basic functions.
5. Evaluate definite and indefinite integrals using substitution.
6. Apply integration to solve work problems involving variable force or distance.
7. Parameterize functions and equations.
8. Solve basic initial-value vector function problems.
9. Evaluate iterated integrals.
10. Convert a Cartesian iterated integral into an equivalent expression in polar coordinates.
11. Set up iterated integrals to calculate center of mass, volumes, and average values.
12. Be able to classify differential equations based on linearity, order, and homogeneity.
13. Use differential equations to model problems involving growth, decay, motion, heating/cooling, and mixing.
14. Find equilibrium solutions to differential equations.
15. Sketch approximate solution curves to 1^{st} order differential equations using slope fields.
16. Use Euler's method to approximate a numerical solution to 1^{st} order differential equations.
17. Use separation of variables to solve basic 1^{st} order ordinary differential equations.
18. Use the characteristic equation to solve basic 2^{nd} order ordinary differential equations.
19. Use eigenvalues and eigenvectors to find solutions to systems of 1^{st} order linear homogenous differential equations.
20. Describe long term behavior of solutions to differential equations.
21. Sketch approximate solution curves to systems of differential equations using phase portraits.
22. Employ recursion relations to determine the coefficients of the power series solution to a differential equation.

MA206 - PROBABILITY & STATISTICS
AY 13-14 - Class of 2016
COURSE DESCRIPTION

MA206 is the final course in the mathematics core curriculum. It provides a professional development experience upon which cadets can structure their reasoning under conditions of uncertainty and presents fundamental probability and statistical concepts that support the USMA core curriculum. Coverage includes data analysis; modeling, probabilistic models, simulation, random variables and their distributions, hypothesis testing, confidence intervals, and simple linear regression. Applied problems motivate concepts, and technology enhances understanding, problem solving, and communication. The course is divided into four blocks of material:

Block 1: Descriptive Statistics and Probability Theory

Objectives:

(1) Be able to display, analyze, and interpret data visually.
(2) Be able to calculate and discuss uses for numerical measures of location and variability.
(3) Understand and use appropriate notation to describe events and probability statements.
(4) Use elementary counting methods to determine the size of certain sets. Find probabilities using these results.
(5) Understand and apply conditional probability, Bayes' theorem, and independence.

Block 2: Random Variables and Empirical Distribution Functions

Objectives:

(1) Be able to define random variables as real-valued functions. Be able to discuss the differences between discrete and continuous random variables.
(2) Understand how to calculate and interpret probabilities, expected values, and variances for discrete and continuous random variables.
(3) Discuss the relationship between a probability mass function (PMF) or probability density function (PDF) and the corresponding cumulative distribution function (CDF). Be able to construct a CDF from a PMF or PDF (and vice-versa) and use either to compute probabilities.
(4) Recognize a binomial experiment and understand the assumptions underlying a binomial experiment. Calculate appropriate binomial probabilities.
(5) Understand what an Empirical Distribution Function (EDF) represents and why it is useful in making inferences about a population; be able to create an EDF by hand and using Microsoft Excel.
(6) Under appropriate conditions, use uniform, normal, gamma, or exponential random variables to model situations. Calculate probabilities as required.

Block 3: Inferential Statistics and Linear Regression

Objectives:

(1) Understand and apply the Central Limit Theorem (CLT).
(2) Interpret the meaning of a con_dence interval. Calculate required confidence intervals for the population mean based on the underlying population distribution, sample data, and sample size.
(3) Construct appropriate null and alternative hypotheses.
(4) Calculate the P-value for tests of the mean of normally distributed populations.
(5) Be able to draw appropriate conclusions from the P-value of a hypothesis test.
(6) Understand the Principle of Least Squares.
(7) Be able to create, use, and interpret a simple linear regression model.
(8) Be able to conduct and interpret results from the Model Utility Test.

Block 4: Linear Regression and Monte Carlo Simulation

Objectives:

(1) Determine the adequacy and appropriateness of a simple linear regression model on a given data set.

(2) Be able to identify situations where regression with transformed variables is more appropriate.

(3) Develop and assess multiple regression models with quantitative and categorical variables.

(4) Understand how statistical simulation helps us solve real world problems and use basic Monte Carlo simulation techniques to solve and interpret real world problems.

Examples of Embedded Skills in MA206 which Support Course Objectives

1. Understand, construct, and interpret visual representations of data, measures of location, and measures of variability.
2. Understand, create and use an EDF to make inferences about a population.
3. Use Microsoft Excel to fit parametric functions to an EDF, and, interpret and compute the error between a functional model (linear or non-linear) and an EDF.
4. Understand the concept of the PDF (the derivative of the CDF).
5. Create a simple, computer-based Monte Carlo Simulation and use the results to answer probability questions.
6. Understand the theoretical/conceptual processes involved with joint PDFs and calculate solutions for joint PDF probability questions.
7. Define a random variable in terms of a real-valued function.
8. Compare and contrast discrete and continuous random variables.
9. Create a probability mass function (PMF) in a table or graphical format.
10. Construct a CDF from a PMF (and vice-versa) and be able to use either to solve probability calculations.
11. Understand the theoretical concept of conditional probability.
12. Understand and calculate the expected value and variance of a discrete random variable.
13. Use basic counting techniques to solve problems involving combinations and permutations.
14. Recognize and understand the assumptions underlying a binomial experiment.
15. Calculate solutions for probability questions involving the binomial and Poisson distributions.
16. Understand and be able to apply the Central Limit Theorem to solve probability problems.
17. Calculate confidence intervals for the population mean based on parameter values, the underlying population distribution, and sample data.
18. Explain the meaning of a confidence interval.
19. Understand hypothesis testing in general terms and be able to construct the proper null and alternative hypotheses for a given problem.
20. Conduct a hypothesis test, understand the relationship between the test statistic and the p-value, and properly interpret the p-value.
21. Understand the difference between predictor (independent) variables and response (dependent) variables.
22. Given a bivariate data set, construct a simple linear regression model.
23. Understand the specific distribution of the random error term in simple linear regression.
24. Understand how the principle of least squares is used to find the best linear model. Modify linear model parameters in order to minimize SSE.
25. Understand the calculation and interpretation of the coefficient of determination (R-squared).
26. Use the slope parameter from a linear regression model to understand the relationship between the variables of interest.
27. Understand the purpose of and execute the model utility test.
28. Create a graph of the residuals of a linear regression model.
29. Create a normal probability plot for a linear regression model.

MA153 – ADVANCED MULTIVARIABLE CALCULUS
AY 13-14 – Class of 2017
COURSE DESCRIPTION

1. MA153 is the first course of a two-semester advanced mathematics sequence for selected cadets who have validated single variable calculus and demonstrated strength in the mathematical sciences. It is designed to provide a foundation for the continued study of mathematics, sciences, and engineering. This course consists of an advanced coverage of topics in multivariable calculus. An understanding of course material is enhanced through the use of *Mathematica* (computer algebra system) and *Excel* (spreadsheet).

2. The course is broken down into four blocks of instruction. Students are expected to become proficient in the block objectives, as well as the course-level technology objectives.

A. Vectors and the Geometry of Space - This block aims to introduce vectors and three-dimensional coordinate systems, ultimately constructing the foundation for the study of multivariable functions. Students will become familiar with vectors and use them to express lines and planes in space. They will represent functions using analytic, numerical, verbal, and graphical methods. They will be able to model and analyze three-dimensional problems incorporating position, velocity, and acceleration.

B. Partial Derivatives of Multi-Variable Functions - This block seeks to develop in the student a solid understanding of rates of change and focuses on the students' ability to incorporate the four methods of function representation introduced in Block 1. The students will expand their single-variable understanding of limits, continuous functions, derivatives, and differentials. They will be introduced to the gradient and will find the maximum and minimum values of functions, with and without constraints.

C. Multiple Integrals - This block will develop the students' understanding of continuous accumulation and will further develop their comfort of graphical expression of multi-variable functions. They will apply this understanding to compute volumes, surface areas, masses, centers of mass, and moments of general regions. Students will express regions in rectangular, polar, cylindrical, and spherical coordinates.

D. Vector Calculus - This block focuses on the study of the calculus of vector fields and serves as a foundation for future mathematics and engineering courses. Students will explore vector fields and be able to apply this knowledge towards solving engineering problems involving different types of fields. Students will extend their understanding of arc length and surface area into the study of line and surface integrals. The block culminates by developing the connection between the Fundamental Theorem of Calculus, double integrals, and triple integrals with the Fundamental Theorem of Line Integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem.

Examples of Embedded Skills in MA153 which Support Course Objectives

1. Determine the vector between two points.
2. Determine the length (magnitude) of a vector.
3. Perform arithmetic operations on vectors.
4. Determine the unit vector in a given direction.
5. Determine the resultant force of several forces.
6. Determine the dot (scalar) and cross (vector) products of two vectors.
7. Determine the angle between vectors, lines, and planes.
8. Determine the scalar projection and vector projection of one vector onto another.
9. Use vectors to compute work.
10. Determine if two vectors are parallel or orthogonal.
11. Use vectors to compute the equations of lines and planes.
12. Determine the point of intersection of a line and a plane.
13. Determine the equation for the line of intersection of two planes.
14. Determine the distance between a point to a plane and two parallel planes.
15. Determine limits, derivatives, and integrals of vector functions.
16. Determine the tangent line to a space curve at any point.
17. Determine arc length and curvature of space curves.
18. Determine the normal and binormal vectors of a space curve.
19. Determine velocity from acceleration and position from velocity.
20. Determine domain, range, limits, and continuity of a function of two or more variables.
21. Compute the partial derivatives of a function of two or more variables.
22. Determine the tangent plane to a surface.
23. Determine the linear approximation to a function of two variables.
24. Determine the total differential of a function of a function of two variables.
25. Determine the directional derivative and gradient vector of a function of two or more variables.
26. Determine the critical points of a function.
27. Determine the absolute maximum and minimum values of a function of two variables.
28. Use the method of Lagrange multipliers to determine the absolute maximum and minimum values of a function.
29. Approximate a double integral using double Riemann sums
30. Evaluate double and triple integrals using iterated integrals.
31. Evaluate a double integral by changing to polar coordinates.
32. Evaluate a triple integral by changing to cylindrical or spherical coordinates.
33. Be able to interpret two- and three-dimensional vector fields.
34. Determine the line integral of a multi-variable function and a vector field.
35. Use Green's Theorem to evaluate a line integral.
36. Compute the curl and divergence of a vector field.
37. Understand the physical significance of curl and divergence.
38. Compute the surface integral over a surface.
39. Compute the surface integral of a vector field.
40. Use Stokes's Theorem to evaluate a surface integral.
41. Understand the basic definitions of a sequences and series.
42. Compute the first *n* terms of a sequence.
43. Determine the formula for the general term of a sequence.
44. Determine whether a sequence converges or diverges.
45. Determine whether a sequence is increasing, decreasing, or not monotonic.

46. Determine whether a sequence is bounded.
47. Compute the n^{th} partial sum of a series.
48. Determine whether a series converges or diverges.
49. Compute the sum of a convergent series.
50. Know the Geometric and Harmonic Series.
51. Determine a power series representation of a function.
52. Differentiate and integrate a given power series.
53. Determine the Taylor or Maclaurin series of a function.
54. Approximate a function using either a Taylor or Maclaurin Polynomial.

MA255 – MATHEMATICAL MODELING & INTRODUCTION TO DIFFERENTIAL EQUATIONS
AY 13-14 – Class of 2017
COURSE DESCRIPTION

1. MA255 is the second course of a two-semester advanced mathematics sequence for selected cadets who have validated single variable calculus and demonstrated strength in the mathematical sciences. It is designed to provide a foundation for the continued study of mathematics, sciences, and engineering. This course emphasizes the interaction between mathematics and the physical sciences through modeling with differential equations. An understanding of course material is enhanced through the use of *Mathematica* (computer algebra system) and *Excel* (spreadsheet).

2. The course is broken down into four blocks of instruction. Students are expected to become proficient in the block objectives, as well as the course-level technology objectives.

A. First and Second Order Differential Equations. This block introduces the concept of differential equations within a modeling framework. Students will create mathematical models with differential equations, conduct qualitative analysis, learn techniques to solve linear differential equations, and gain some understanding of the history of this important branch of mathematics. Students will use second order linear equations with constant coefficients to model a variety of mechanical and electrical vibration problems. Students will continue to use technology to graph solutions.

B. Second Order Linear Equations and Series Solutions. This block begins with a continuation of Chapter 3 (second order ODEs)—including modeling with forcing functions. This block also introduces series as a means to approximate the value of functions. The student will become familiar with the basic concepts of sequences and infinite series, as well as different convergence tests. Power series are introduced as a means to approximate functions and the student is expected to be able to approximate a function using either a Taylor polynomial. Finally, students will solve differential equations through series solutions.

C. Laplace Transform and Systems of First Order Linear Equations. This block consists of two parts: Laplace transforms and systems of first order linear equations. Chapter 6 will introduce the Laplace transform method in solving linear differential equations, emphasizing problems typical in engineering applications. Chapter 7 will introduce theory and solution techniques for solving first order linear systems.

D. Numerical Solutions to Differential Equations and Nonlinear Differential Equations. This block introduces numerically solving differential equations through Euler's Method, the Improved Euler's Method, and Runge-Kutta method. We will investigate systems of nonlinear systems of ordinary differential equations using analytical and qualitative techniques. Students will build on their existing *Mathematica* & *Excel* knowledge by finding exact solutions to first order differential equations, as well as use numerical methods to approximate solutions.

Examples of Embedded Skills in MA255 which Support Course Objectives

1. Derive differential equations that mathematically model simple problems.
2. Set up and solve initial value problems.
3. Apply Euler's method to solve first order initial value problems.
4. Classify differential equations with respect to order and linearity.
5. Compute the integrating factor and general solution for linear differential equations.
6. Determine the general solution for a separable equation.
7. Understand the three identifiable stages that are always present in the mathematical modeling process:
    a. Construction of the model.
    b. Analysis of the model.
    c. Comparison with experiment or observation.
8. Determine the qualitative behavior of solutions to autonomous equations.
9. Solve exact differential equations.
10. Use integrating factors to convert a differential equation that is not exact into an exact equation and solve.
11. Solve a system of two linear algebraic equations.
12. Understand and apply the properties of matrices and matrix algebra.
13. Find the eigenvalues and eigenvectors of a matrix.
14. Sketch the phase plane and phase portraits of a 2×2 linear system.
15. Classify the critical point of a system of two linear differential equations with respect to type and stability.
16. Construct the solution to a system of two linear differential equations.
17. Determine equilibrium solutions for an autonomous system of two linear differential equations.
18. Transform a second order differential equation into a system of first order equations.
19. Sketch the phase portrait of a nonlinear system.
20. Solve a linear homogeneous initial value problem.
21. Construct the initial value problem for mechanical and electrical vibrations.
22. Construct the solution to a second order non-homogeneous differential equation.
23. Use the definition of the Laplace transform to compute the Laplace transform of continuous, piecewise continuous, discontinuous, and periodic functions.
24. Determine the inverse Laplace transform of a function using a table of elementary Laplace Transforms.
25. Determine the solution to an initial value problem for a second order linear differential equation using Laplace transforms.
26. Determine the solution to an initial value problem for a system of two linear equations using Laplace transforms.
27. Transform an n^{th} order differential equation into a system of first order equations.
28. Solve a system of linear differential equations with real and complex eigenvalues.
29. Compute and classify the critical points of a nonlinear autonomous system.
30. Compute the corresponding linear system near the critical points of a nonlinear autonomous system.
31. Construct the initial value problem for competing species and predator prey problems.
32. Use phase plane methods to investigate solutions to competing species and predator prey models.

MA364 - ENGINEERING MATHEMATICS
AY 13-14 - Classes of 2015 and 2016
COURSE DESCRIPTION

1. MA364 provides additional mathematical techniques and deepens the understanding of concepts in mathematics to support continued study in science and engineering. Emphasis is placed upon using mathematics to gain insight into natural and man-made phenomena that give rise to problems in differential equations and vector calculus. Calculus topics focus on three-dimensional space curves, vector fields and operations, divergence and curl, line and surface integrals. Analytic and numerical solutions to differential equations and systems of differential equations are found using a variety of techniques. Linear algebra topics include solutions to homogeneous and non-homogeneous systems of equations. An introduction to classical partial differential equations is included in the spring semester.

Objectives at the topical level are:

A. Vector Calculus
   (1) Parameterize curves and surfaces in 2 and 3-space.
   (2) Calculate the gradient and understand it as a tool for measuring the local rate of change (in magnitude and direction of steepest ascent) in a scalar field. Apply the gradient to potential (conservative) fields, as necessary.
   (3) Calculate the divergence and understand it as a measure of outward/inward flow or expandability/compressibility.
   (4) Calculate the curl and understand it as a measure of the local rotation (circulation) at a point in a vector field.
   (5) Set up and evaluate line integrals to measure work and circulation.
   (6) Understand the relationship between path-independent line integrals, exact differential forms, and conservative fields. Apply the Fundamental Theorem of Line Integrals.
   (7) Use Green's theorem to convert between line integrals and double integrals.
   (8) Set up and evaluate surface integrals to measure the flux through surfaces.
   (9) Understand the Divergence Theorem and use it to evaluate flux integrals.
   (10) Use Stoke's Theorem to convert between line integrals and surface integrals. Understand the relationship between Green's and Stoke's theorems.

B. Ordinary Differential Equations
   (1) Understand the geometry of the complex plane and the relationship between rectangular and polar representations of complex numbers.
   (2) Perform arithmetic with complex numbers, including multiplication, division, powers, roots, and exponentiation.
   (3) Classify ordinary differential equations by order, linearity, homogeneity, and coefficient type.
   (4) Model physical systems, such as exponential decay or spring/mass systems, with ordinary differential equations.
   (5) Solve second-order linear constant coefficient homogeneous differential equations using the characteristic equation. Understand how complex roots of the characteristic equation lead to real-valued solutions to the differential equation.
   (6) Know the three conditions for and the behavior of the three types of damping: under, critical, and over, for a spring/mass system.
   (7) Understand and apply the unit step (Heaviside) and Dirac Delta functions in modeling physical systems with discontinuous inputs.
   (8) Solve second-order ordinary differential equations with continuous and discontinuous inputs using Laplace transforms.

(9) Solve linear systems of ordinary differential equations.

C. Fourier Series and Partial Differential Equations

(1) Express periodic functions as linear combinations of sine and cosine functions using Fourier series. Conversely, understand the convergence of the Fourier series to the periodic extension of the function.

(2) Understand the concepts related to orthogonal functions.

(3) Construct even and odd extensions to functions, and represent those extensions as linear combinations of sine and cosine functions using Fourier sine or cosine series.

(4) Understand Fourier series as an expansion in terms of sine and cosine basis functions, much like the expansion of three dimensional vectors in terms of the unit basis vectors **i**, **j**, and **k**.

(5) Understand the concepts of a partial differential equation containing boundary conditions, and, in particular, the construct of the heat equation.

(6) Understand the construct of solutions pertaining to the heat equation. Solve the one-dimensional heat equation for various boundary conditions.

2. Develop cadets' modeling abilities by regularly demonstrating and applying these new mathematical tools to simple, intuitive applications.

3. Develop cadets' proficiencies in the problem solving process by:

– Formulating equations and performing analysis of more complex models with technology.

– Applying advanced specialized concepts to important scenarios which are not easily analyzed using by-hand techniques, and interpreting their solutions into observations, predictions, and recommendations in the context of the original scenario.

INTERDISCIPLINARY GOAL
AY 13-14

The Department of Mathematical Sciences believes that cadets learn mathematics by doing mathematics. Traditionally, this has meant daily work that enables cadets to practice and explain their problem-solving skills, which enables instructors to gauge the cadet's understanding. While this practice continues, the advent of calculators, computers, and associated software now provides opportunities for cadets and teachers to create and explore more sophisticated problems. In developing these "more sophisticated problems" mathematics instructors here, as elsewhere, have naturally turned to mathematical applications in nature, science, engineering, economics, as well as to other fields including physical education and psychology. As a result, the mathematics faculty is often engaged in conversations dealing with how to best prepare cadets to wield these new weapons of discovery and learning. We started this sort of learning model with the Interdisciplinary Lively Applications Project (ILAP) program in AY 92-93 and have included projects in our courses since. This year, we added an interdisciplinary perspective goal to our core program goals in an effort to ensure interdisciplinary efforts are emphasized throughout the program and to encourage further collaboration with other departments in this endeavor.

Interdisciplinary projects are valuable as they:

- excite cadets with the power of mathematics as a tool in describing, investigating, and solving applied problems.
- excite cadets about the fields that become accessible to them as they master more mathematics.
- acquaint downstream departments with their future customers, allow these departments to demonstrate the utility of mathematics within their discipline, and involve these departments in exciting these cadets at an early stage of their development.
- enhance inter-departmental cooperation and make the four-year cadet experience more cohesive.

Usually, each core math course presents from one to three projects during the course of the semester that demonstrate realistic applications of the mathematics being studied. In some cases, when the project is assigned to the class, the application department presents an introductory lecture (or video) explaining the project. When the project has been completed, the application department returns to present a concluding lecture (or video) reviewing the formulation and problem solving process, highlighting the important concepts being used, and illuminating extensions of the problem in their discipline. Interdisciplinary projects that have been used in the core mathematics courses include the following:

MA103:

1-D Heat Transfer in a Bar – D/CME (Thermo) – 9301
Pollution Levels in Lake Shasta – D/GEnE – 9301
Tank Battle Direct Fire Simulation – D/SE – 9301
Pollution Levels in the Great Lakes – D/GEnE – 9401/9402
Smog in the LA Basin – D/Chem – 9401
Car Financing – D/SocSci (Econ) – 9402
Pollution Levels in the Great Lakes – D/GEnE – 9501
Car Financing – D/SocSci (Econ) – 9501
Making Water in Space – D/Chem – 9701
Brusselator – D/Chem – 9801
Bioaccumulation of methylmercury – D/Chem – 0001
Civil War –D/History – 0201
College Tuition – 0401
Hot Coffee – Newton's Law of Cooling – 0501
Game Theory – Chutes and Ladders – 0601
Thrift Saving Plan – D/SocSci – 0701
Diabetes – 0801

Use Matrix Algebra to Solve Stoichiometry Problems – D/Chem – 1201
Model Heat Transfer and Tent Insulation – D/Chem – 1301
Analyze Generator Power Requirements for Water Purification – D/Chem – 1301
Essay on How to Model the NetZero Energy Goal at West Point – D/Engl – 1301
Fielding a Soldier Power System – D/Chem, Engl, BS&L, EECS – 1301

MA104:

Aircraft Ranges under Various Flight Strategies – D/CME (Aero) – 9302
From Bungee Cords to the Trebuchet – D/CME (Vibes) – 9302
Oxygen Consumption and Lactic Acid Production – DPE – 9402
From Wing Resonance to Basketball Rims – D/CME (Vibes) – 9402
Patient Scheduling & Profit Management – D/SocSci (Econ) – 9502
Car Suspension System – D/C&ME (Vibes) – 9502
Fighting Forest Fires – D/GEnE – 9501/9502
Flow Rates of A Reservoir Over a Dam – D/CME – 9501/9502
Parachute Jumping – D/Physics – 9501/9502
Modeling Engineer Operations in a Nation Building Scenario – D/CME – 9701/9702
Cut/Fill and Bridge Abutment/Span Computation – D/C&ME – 9701/9702
Railway Headwall Design – D/CME – 9701/9702
Water Contamination and Treatment – D/GEnE – 9801/9802
Forest Fire / Water Tank Properties – D/CME – 9901/9902
Earthquake Analysis of Water Tank – D/CME – 9901/9902
Dissolved Oxygen – D/GEnE – 0101/0102
Determining Speed and Distance of a Motorcycle Jump – D/EECS – 0202
Modeling Tree Foliage – 0302
Aquaculture—This Investment Smells Fishy… - D/GEnE, D/Sosh 0402
Light Refraction and Fermat's Principle in Action – D Physics 0602
Time Makes Money – D/Sosh 0901
Financial Operations Modeling for a DCA Coffee Shop - 1101
Beverage Profit Modeling for the West Point Golf Course Snack Bar - 1102
Chemical Manufacturing Optimization - 1201
Optimizing DCA Tailgate Profits via Sales and Advertising - 1202

MA205:

Vehicular Collisions – D/Physics – 9501
Aircraft Utilization – D/SE – 9501
Airborne Parachutists – D/Physics – 9502
Patient Scheduling & Profit Management – D/SocSci (Econ) – 9502
Tank Production – D/SocSci (Econ) – 9601
Parachute Panic – D/Physics – 9601
Math, the National Pastime? (Baseball) – D/Physics – 9701
Cobb–Douglas Problem – D/SocSci (Econ) – 9701
LAPES (Drop Zone) Problem – D/Physics – 9701
Shuttle Problem – D/Physics – 9801
Satellite Problem – D/CME – 9801
Vector analysis of truss members – D/CME – 9901
Modeling Bangalore Production (DDS /SV &MV Calculus) – D/SocSci (Econ) – 9901
MTA Metrorail Connections – D/Soc Sci (Econ) – 9902
Storage Tank Optimization – D/GEnE – 9902
Flight of a TOW Missile – D/Physics – 0001
Least Squares Regression of Data – D/Physics – 0001

Counterbattery Target Acquisition – D/Physics – 0101
Modernizing Camera Product Lines – D/SocSci (Econ) – 0101
Incoming Interception in BMD – D/Physics – 0201
Treating Motor Pool Runoff – D/GEnE – 0201
Modeling the Spread of Information and Influence - 1001
Modeling the Growth of an Insurgency - 1101
Analyzing Opium Trade in Afghanistan - 1201
Tidal Power -1301
Wind Turbine Analysis - 1301
Modeling Drug Assimilation is the Bloodstream - 1302
Modeling the Spread of Disease - 1302

MA206:
Pollution Levels in the Great Lakes – D/GEnE – 9501/9502
Statistical Analysis of Hudson River Pollution Data – D/GEnE – 9801
Markov Chain Model of Dow Jones Industrial Average – D/SocSci– 9802
Statistical Analysis of Elementary Circuits – D/Physics – 9802
Call for Artillery – D/EECS – 9801
The Dam Problem – D/CME – 9902
Capacitors in Circuits – D/Physics – 0802
Price Controls and Gasoline in Iraq – D/SocSci - 0901

The interdisciplinary goal embodies the belief that early mastery of mathematics skills produces in cadets the realization that they can indeed formulate and analyze interesting problems arising in engineering, science, business, and many other fields. Our experience is that this realization increases cadet motivation. We and the application departments have seen benefits of this increased motivation as cadets move from core mathematics into their disciplines.

Our interdisciplinary material is public domain – any reader interested in these materials can find many of the projects at the Department of Mathematical Science's web page (www.dean.usma.edu/math/), or can contact the core program director for more information.

Beginning in AY2012-2013, the focus of our interdisciplinary work will be built around the theme of energy. The math department has been part of the leadership team in an Academy initiative involving interdisciplinary collaboration. This effort began with participation as one of 25 institutions in a Project Kaleidoscope (PKAL) initiative to enhance interdisciplinarity within core programs. The West Point team chose the topic of "Energy" for preliminary efforts as it is both a timely real world issue and ties into the designation of West Point as a "Net Zero" installation. The department is participating as part of the Academy Core Interdisciplinary Team (CIT) with 8 departments to include energy themed work within the core courses so that students explore the topic from multiple perspectives and see ties between those disciplines in the context of one important issue.

LIAISON PROFESSORS

"Servicing What We Sell"

Most departments at USMA rely on at least some portion of the core mathematics program to prepare cadets for studies in their Department. Additionally, the Department of Mathematical Sciences has found that the study of mathematical principles and methods is often made easier by considering problems in applied settings with realistic scenarios. In order to facilitate success in both of these areas, the Department of Mathematical Sciences has instituted a liaison program in which tenured faculty members serve as Liaison Professors, primarily responsible for coordination with a particular client Department. The designated Liaison Professors for this academic year are as follows:

DEPARTMENT	LIAISON PROFESSOR	PHONE	OFFICE
BS&L (Human Factors)	COL Andy Glen	x5988	TH246B
Chemistry & Life Science	COL Jerry Kobylski	x5608	TH225
C&ME	LTC Tony Johnson	x7685	TH226C
EE&CS	LTC Doug McInvale	x4544	TH255A
GEnE	LTC Paul Goethals	x5619	TH254
Physics	Fr. Gabe Costa	x5625	TH235A
	COL Tina Hartley	x2276	TH239A
Soc Sci (Econ)	LTC Donald Outing	x7217	TH235B
Systems Eng	LTC Doug McInvale	x4544	TH255A

COL Tina Hartley has primary responsibility for oversight of the Liaison Professor program.

The major focus of this program is to achieve a more integrated student MSE experience by promoting coordination and collaboration between the Department of Mathematical Sciences and the other MSE Departments. The role of the Liaison Professor is to serve as the principal point of contact for members of the client department. The Liaison Professor fields questions, accepts suggestions, and works issues from the client department with respect to course material, projects, procedures, timing, and any other matters of mutual interest. Additionally, the Liaison Professor is a first point of contact for course directors in the Department of Mathematical Sciences who are looking for examples and applications appropriate to their course and need referrals.

This program is not intended to preclude closer coordination (for instance, at the course director or instructor level), but rather is intended to provide a continuing source of information and input at the senior faculty level who can ensure that inter-departmental cooperation is accomplished across several courses in a consistent fashion, as required.

UNITED STATES MILITARY ACADEMY
MATHEMATICAL RECALL KNOWLEDGE

The following constitutes a basic mathematical vocabulary that will be built upon during each cadet's four-semester core mathematics experience and in his or her future math/science/engineering courses. Once each of these basic ideas has been covered in class, each cadet can be required to reproduce, upon demand in any future lesson of any math/science/engineering course, that idea exactly as shown here. Annotated beside each heading or item is the course number in which the cadet is responsible for each item . These items are recall knowledge - cadets are also required to be proficient in the more conceptual, less-verbatim ideas and skills reflected in each core math course's Course Objectives section of this document.

ALGEBRA (103/1)

1. $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. $a^b \cdot a^c = a^{b+c}$
3. $(a^b)^c = a^{bc}$
4. $\dfrac{a^b}{a^c} = a^{b-c}$
5. $y = \log_b x \Rightarrow x = b^y$
6. $\log_b b^x = b^{\log_b x} = x$
7. $\log_b x^a = a \log_b x$
8. $\log_b ac = \log_b a + \log_b c$
9. $\log_b \dfrac{a}{c} = \log_b a - \log_b c$
10. $\log_b a = \dfrac{\log_c a}{\log_c b}$

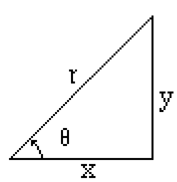
ANALYTIC GEOMETRY (103/1)

Rectangle:	Area = lw	Perimeter = $2l + 2w$
Circle:	Area = πr^2	Circumference = $2\pi r$
Rectangular Solid:	Volume = lwh	Surface Area = $2lw + 2lh + 2hw$
Cylinder:	Volume = $\pi r^2 l$	Surface Area = $2\pi r^2 + 2\pi rl$
Sphere:	Volume = $\frac{4}{3}\pi r^3$	Surface Area = $4\pi r^2$

Distance between (x_1, y_1) and (x_2, y_2) = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

TRIGONOMETRY (103/1)

With reference to the right triangle:



2π radians = 360 degrees

$\sin\theta = \dfrac{y}{r}$	$\cos\theta = \dfrac{x}{r}$	$\tan\theta = \dfrac{y}{x}$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$x^2 + y^2 = r^2$	$\sin^2\theta + \cos^2\theta = 1$
$\cot\theta = \dfrac{1}{\tan\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$	$\csc\theta = \dfrac{1}{\sin\theta}$

RELATIONSHIPS (MA103/1)

Corresponding sides of similar triangles are proportional

Distance = average rate × time

DIFFERENTIATION (MA 104)

1. $\frac{d}{dx}(a) = 0$ (104)
2. $\frac{d}{dx}(x) = 1$ (104)
3. $\frac{d}{dx}(au) = a\frac{du}{dx}$ (104)
4. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$ (104)
5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ Product Rule (104)
6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ Quotient Rule (104)
7. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$ Power Rule (104)
8. $\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$ Chain Rule (104)
9. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$ (104)
10. $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$ (104)
11. $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$ (104)
12. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ (104)

INTEGRATION (MA 205)

13. $\int a\, dx = ax + C$ (205)
14. $\int (u + v)\, dx = \int u\, dx + \int v\, dx$ (205)
15. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ (205)
16. $\int e^{ax}\, dx = \frac{e^{ax}}{a} + C$ (205)
17. Understand and be able to apply the Substitution Rule (205)
18. $\int \frac{du}{u} = \ln |u| + C$ (205)
19. $\int \sin (ax)\, dx = -\frac{1}{a}\cos (ax) + C$ (205)
20. $\int \cos (ax)\, dx = \frac{1}{a}\sin (ax) + C$ (205)
21. If f is integrable on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$ where $\frac{dF}{dx} = f(x)$

    (Fundamental Theorem of Calculus) (205)

VECTOR CALCULUS (MA104)

22. $|\vec{A}| = \sqrt{a_i^2 + a_j^2 + a_k^2}$ (104)
23. $\vec{A} \bullet \vec{B} = a_i b_i + a_j b_j + a_k b_k = |\vec{A}||\vec{B}|\cos\theta$ (104)
24. $|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|\,|\sin\theta|$ (104)
25. $\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$ (104)

PROBABILITY & STATISTICS (MA 206)

26. PDFs and CDFs: (206)

Discrete: PDF: $P(X = x) = p(x)$.

CDF: $P(X \le x) = \sum_{y \le x} p(y)$.

Continuous: PDF: $f(x)$ is used to find probabilities - $P(a \le X \le b) = \int_a^b f(x)dx$.

CDF: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy$.

27. The total accumulation of a probability distribution function is 1. (206)

Discrete: $\sum_{\forall x} p(x) = 1$ Continuous: $\int_{-\infty}^{\infty} f(x)dx = 1$

28. Calculate and interpret the expected value (mean) of a random variable. (206)

Discrete: $E(X) = \sum_{\forall x} x \cdot p(x)$ Continuous: $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$

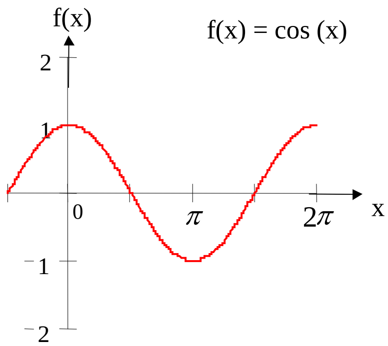
29. Calculate and interpret the variance of a random variable. (206)

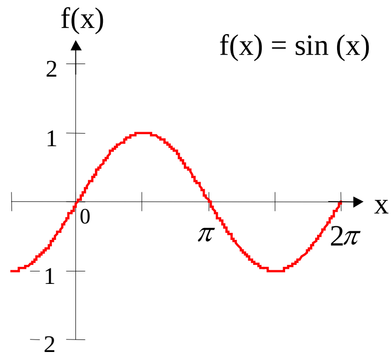
The variance is an expected value: $V(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$.

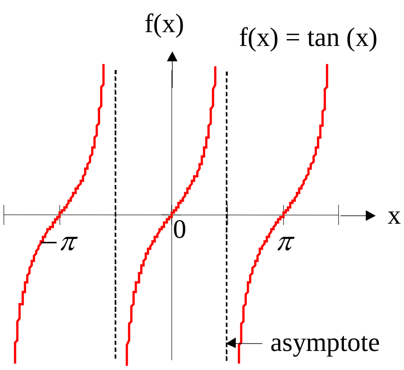
30. Percentiles of random variables including the median (50^{th} percentile). (206)

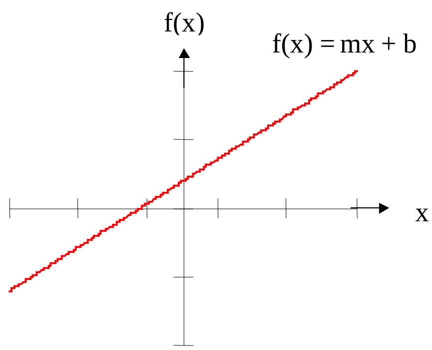
31. Central Limit Theorem: Let $X_1, X_2, \ldots, X_n$ be a random sample from *any* population with a finite mean and variance. Then, if *n* is sufficiently large, $\overline{X} = \sum X_i / n$ and $T_0 = \sum X_i$ are **approximately** normally distributed. (206)

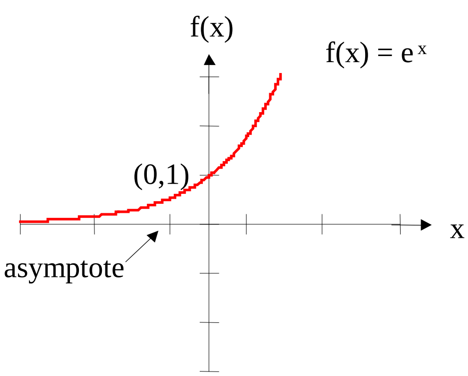
PROPERTIES OF FUNCTIONS (103/1)

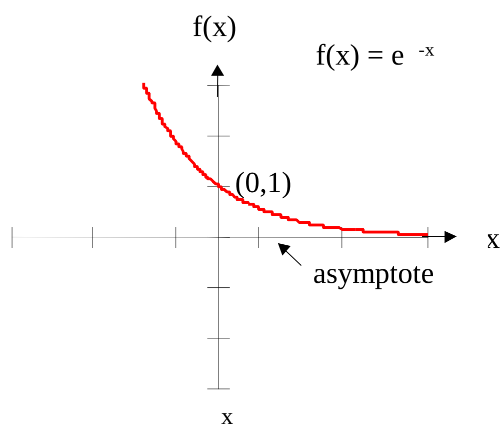


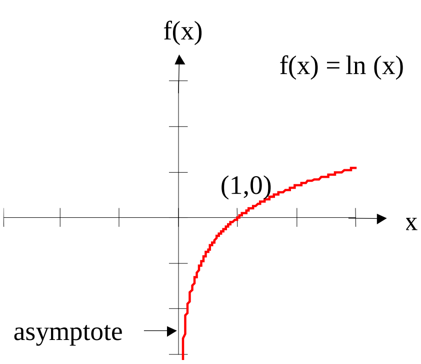












UNITED STATES MILITARY ACADEMY
REQUIRED SKILLS IN SCIENTIFIC COMPUTING

The following list constitutes the basic, required scientific computing skills that you will learn and use during your four-semester core mathematics experience and in your future math/science/engineering courses. Once each of the listed topics is covered in class, you must be able to execute this skill upon demand in any future lesson or course. Reference books, for the respective computing system, are the chief source of information for learning these skills. Annotated beside each item is the course, which denotes the exact point in time you are responsible for that particular item (e.g. 103 indicates responsibility for that skill following MA103.

LAPTOP SKILLS (MATHEMATICA and MS EXCEL)

The following are all laptop (or desktop) computer skills. Most can be accomplished in either Mathematica or Excel, but some are easier in one package. As you become proficient in each skill, it is your job to decide which tool is the right one for the task at hand.

1. Enter a recursive relationship and evaluate analytically, numerically and graphically:

   E.g. Given the difference equation $a_{n+1} = 3a_n + n^2$ (103)

   a. Find a_{100}.

   b. Display a table of "n" and "a_n" values.

   c. Graph the time-series plot of a_n.

2. Perform basic matrix and vector operations
   a. Given 2 matrices, compute (if defined) their sum and product. (103)
   b. Find the inverse of a given square matrix (if it exists). (103)
   c. Find the eigenvalues and eigenvectors of a given matrix. (103/205)
   d. Find the magnitude of a given vector. (104)
   e. Given 2 vectors, compute their dot product and cross product. (104)

3. Perform matrix operations on linear systems of equations in two and three variables:

   E.g. Given the linear system of equations, $\begin{bmatrix} 2 & 5 & 6 \\ 7 & 8 & 4 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$

   a. Solve the system of equations. (103)
   b. Perform row reduction operations on the augmented matrix. (103)

4. Use the *Solve* or *FindRoot* commands in *Mathematica* to find the roots of a polynomial or transcendental function (both real and complex) or to solve simultaneous equations.

   E.g. a. Find the roots of the function $r^3 - 7r^2 - 14r + 8 = 0$ (103)
   b. Find the roots of the function $e^x - 2 = 0$.
   c. Solve to find the point of intersection between $2x + 3y = 7$ and $y - x = 4$.

5. Define and evaluate a function at a specific point in *Mathematica* and *Excel.* (103)
   E.g. $f(x) = 2x^2 + 3x - 24e^{2x}$; $g(x) = \sin(3x) + e^x - 2\ln(x)$

6. Plot a function or several functions over a specified domain. (103)

7. Use the *Excel* Solver to minimize the squared error when fitting models to data. (103)
   Minimize the SSE between known distributions and the EDF. (206)

8. Plot 3-dimensional objects (surfaces, space curves, and level curves):

   E.g. Plot the space curve (104)
   $\vec{r}(t) = t^2\,\vec{i} + 2t\,\vec{j} + \sin(t)\,\vec{k}\,,\, t \in [0, \pi]$

   E.g. Plot the surface $z = \dfrac{x^2}{4} + \dfrac{y^2}{16}$. (104)

   E.g. Plot the level curves for $x^2 + y^2 = k$ (104)

9. Evaluate limits: (103/104)

   E.g. Evaluate $\lim\limits_{x \to 5} \dfrac{x-5}{x^2 - 25}$.

   E.g. Evaluate $\lim\limits_{x \to \infty} \dfrac{2\sqrt{x} + x^{-1}}{3x - 7}$.

10 Evaluate derivatives:

   E.g. Given the function $y = \dfrac{1}{x} + \sin(2x^3\text{-}x)$,

   a. Find the derivative. (104)
   b. Plot the derivative. (104)

   E.g. Given the function $f(x, y) = \dfrac{1}{y} + \sin(2x^3 - y)$, find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$. (104)

   E.g. Given the function $f(x, y) = x^2 y + xy^2$,

   a. Find the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial^2 f}{\partial x \partial y}, \dfrac{\partial^2 f}{\partial x^2}$. (104)

   b. Find the gradient of $f(x,y)$ at the point (2,3). (104)

   c. Find the directional derivative of $f(x,y)$ at the point (2,3) in the direction of $2\vec{i} - 4\vec{j}$. (104)

11. Solve nonlinear systems of equations:
    a. Solve for relative extrema (104)
    b. Constrained optimization (Lagrange) (104)

12. Calculate Riemann sums to approximate net change. (205)

13. Evaluate definite and indefinite integrals:

    E.g. Evaluate $\int (x \ln(x) - e^x)dx$ (205)

E.g. Evaluate $\int_0^{\pi}\left(x^2+\cos(2x)\right)dx$. (205)

E.g. Evaluate $\int_0^3\int_0^2\left(x^2 y\right)dy\, dx$ (205)

E.g. Evaluate $\int_0^1\int_0^{y^2}\left(3y^3 e^{xy}\right)dx\, dy$. (205)

E.g. Evaluate $\int_0^3 \sin\left[\frac{(2n-1)\pi\, x}{6}\right]\sin\left[\frac{(2m-1)\pi\, x}{6}\right]dx$, (364)
for $m = n$ and $m \neq n$.

14. Calculate distance traveled along a curve using the concept of arc length. (205)

15. Plot surfaces and regions of integration. (205)

16. Differential Equations (DE):

E.g. Given the DE $\frac{dy}{dx} = \cos(\pi x) + e^{-x} - 2$,

a. Plot the direction (slope) field of the DE. (205)
b. Graphically approximate a solution curve to the DE for $y(0) = 2$. (205)
c. Use Euler's Method to find a numerical approximation for $y(3)$ given $y(0) = 2$. (205)
d. Verify a given solution to the DE (205)

17. Probability and Statistics:

a. Fit a regression line to given data by minimizing the SSE. (104, 206)
b. Compute sample statistics from a data set. (206)
c. Plot a histogram. (206)
c. Create an empirical CDF from a given matrix of data. (206)
d. Analyze data with descriptive statistics and descriptive plots. (206)
e. Integrate known distributions to determine probabilities. (206)
f. Calculate expected value and population variance using integration. (206)
g. Calculate the inverse CDF using FindRoot or Solve commands. (206)
. h. Simulate data (generate pseudo-random data) (206)
i. Perform linear regression and interpret the output (206)

18. Seek on-line HELP to learn new commands, uses, and methods, as well as to troubleshoot one's own efforts.

MS-WORD, with Graphics Objects from other Packages

19. Create a formal document in electronic form that includes textual, graphical, numerical, and analytical modeling and analysis. (103)

REQUIRED MATHEMATICAL SKILLS FOR ENTERING CADETS

All students entering the core academic program at West Point should possess a core set of math skills in order to effectively advance their learning, especially in the required math, science, and engineering courses. Our experience indicates that if students lack a good understanding of these concepts, then they will most likely not perform to their full potential. The MSE Committee deems the items below to be fundamental skills and concepts that all entering cadets must possess. To assess a student's knowledge of the fundamental skills listed below, the Department of Mathematical Sciences administers a Fundamental Concepts Exam (FCE), also known as the Summer Gateway Exam, to all new cadets during Cadet Basic Training. We encourage incoming cadets to evaluate their mathematics skills and work at remediation of any deficiencies prior to arriving at USMA.

The Department of Mathematical Sciences also administers three additional Fundamental Concepts or "Gateway" Exams during the first semester in MA103 (Discrete Dynamical Systems and Introduction to Calculus). Self-paced texts and software files offer opportunities for self-remediation. Failure to pass at least one "Gateway" exam at mastery level (≥ 80%) may result in placement in MA100 (Precalculus) or reenrollment in MA103 during the second semester instead of continuing into MA104 (Calculus). Cadets enrolled in MA100/101 who do not demonstrate proficiency in these fundamental mathematics skills by the end of their first year may be separated from USMA.

Almost any high school algebra book is a suitable reference for these fundamental skills. An incoming cadet can obtain additional exam resources by going to the following link: http://www.westpoint.edu/math/SitePages/Prospective Students.aspx.

Note: All calculations must be done <u>without</u> the use of technology (i.e., calculator). Some examples of skills are provided in parentheses.

1. Algebra and Real Numbers.

a. Use symbols and operators to represent ideas and objects and the relationships existing between them.

b. Understand the relationship between measures of the physical world. (Velocity, distance and time: On a 40-mile car trip to Middletown, NY, you drive the first twenty miles at 40 mph and the last twenty miles at 60 mph. What is your average speed during the trip?)

c. Know and apply the following algebraic properties of the real number system: identity, associative, commutative, inverse, and distributive.

d. Express numbers using scientific notation. (Express 0.004312 in scientific notation.)

2. Radicals and Exponents.

a. Convert between radical and rational exponent form. (Transform $\frac{1}{\sqrt{x+2}}$ to the rational exponent form $(x+2)^{-\frac{1}{2}}$.)

b. Manipulate algebraic expressions that contain integer and rational exponents. (Simplify $4^{-\frac{3}{2}} \cdot 27^{-\frac{2}{3}}$.)

3. Algebraic Expressions.

a. Add, subtract, multiply, and divide algebraic expressions. (Find the remainder when $x^3 - 7x^2 + 9x$ is divided by $x - 2$.)

b. Simplify algebraic expressions. (Expand and simplify $(x\text{-}3)(x\text{-}2)(x\text{-}1)$.)

4. Factoring / Prime Numbers

a. Write a number as the product of factors. (Write 42 as the product of prime factors.)

b. Solve for the roots of a polynomial by factoring. (Find the roots of $x^2 - 5x + 6 = 0$.)

5. Linear Equations, Inequalities and Absolute Values.

a. Solve 2 simultaneous linear equations by graphing and by substitution. (Use a graph to estimate the point of intersection of the lines $2x + 3y = 7$ and $-x + y = 4$. Verify your result using back substitution.)

b. Solve linear equations and inequalities [graphically and algebraically]. (Solve $5(3-x) > 2(3-2x)$ for x.)

c. Solve linear equations and inequalities with absolute values. (Solve $| x - 4 | \geq 3$ for x.)

6. Polynomials and Rational Inequalities.

a. Solve simple polynomial inequalities. (Solve $x^2 + 3x + 6 > x - 4$ for x.)

b. Solve simple rational inequalities. (Solve $\frac{x-3}{x+1} < 2$ for x.)

7. Straight Lines.

a. Determine the equation of a line. (Find the equation of a straight line passing through the points (2, 1) and (5, 4).)

b. Determine the equation of a line that is parallel or perpendicular to a given line. (Find the equation of a line parallel to the line $2y - 3x = 7$ and passing through the point (1, 2).)

8. Functions.

a. Identify the independent and dependent variables of a function.

b. Determine the domain and range of a real valued function. (Find the domain and range of the real valued function $g(x) = \frac{1}{x^2 - 2}$.)

c. Evaluate a function at a point. (Given $f(x) = 3x^2 - 2x + 4$, find $f(2a)$.)

d. Evaluate composite functions. (Given $h(r) = 3r^2$ and $g(s) = 2s$, find $h(a+2) - g(2a)$.)

9. Quadratic Equations and Systems

a. Solve for real and complex roots using the quadratic formula. (Find the roots of $3x^2 + 2x = -1$.)

b. Solve a system of quadratic equations in 2 variables by substitution. (Solve the system $y = 3-x^2$ and $y = 4 + 2x^2 - 2x$.)

10. Trigonometric Functions.

a. Define each of the 6 trigonometric functions ($\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, $\csc(\theta)$) in terms of the sides of a right triangle. ($\cos(\theta) = x/r$ where x is the adjacent side and r is the hypotenuse.)

b. Define each of the 6 trigonometric functions in terms of $\sin(\theta)$ and $\cos(\theta)$. ($\tan(\theta) = \frac{\sin\theta}{\cos\theta}$.)

c. Know the domain and ranges for the sine, cosine, and tangent functions.

d. Convert angle measures between degrees and radians. (Write 120 degrees as a radian measure.)

e. Memorize and use the 30/60/90 and 45/45/90 degree reference triangles.

f. Know and apply the trigonometric identity $\sin^2(\theta) + \cos^2(\theta) = 1$. (Simplify the expression $2\cos^2(\theta) + \sin^2(\theta) - 1$.)

11. Logarithmic and Exponential Functions.

a. Know the relationship between logarithm and exponential functions [$y = \log_a x$, $a>0$, $a\neq1$, is the inverse of the function $y = a^x$; $\log_a x = y \Leftrightarrow a^y = x$)] (Evaluate $\log_3 27$.)

b. Know the properties of the logarithmic and exponential functions and use them to simplify logarithmic expressions. (Express as a single logarithm: $.5 \log_{10} x - \log_{10} y$.)

c. Solve simple logarithmic and exponential equations. (Solve the equation $3^{x+4} = 4$. for x.)

12. Graphs and Graphing.

a. Graph equations and inequalities. (Sketch a graph of the function $f(x) = 3x^2 - 2x + 7$ for $1<x<5$.)

b. Properly label a graph (axes, intercepts, asymptotes, and roots).

c. Know the general characteristics and shapes of the graphs of polynomial, logarithm, exponential and trigonometric functions.

d. Transform the graph of a known function. (From the graph of $f(x)$, graph the function $g(x) = 2f(x) - 3$.)

13. Analytic Geometry.

a. Know and apply the distance formula between 2 points. (Find the distance between the two points A(1,2) and B(-5,-3).)

b. Know and apply the circumference and area formulas for circles, triangles, and rectangles. If you double the radius of a circle, what happens to its circumference?)

c. Know and apply the surface and volume formulas for cylinders, spheres and rectangular solids.

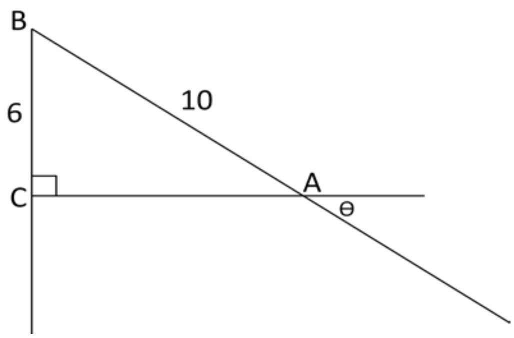
d. Know the relationship between similar triangles. (A rectangle with base x and height 5 is inscribed in an isosceles triangle with base 10 and height 20. Determine x.)

e. Know and apply the Pythagorean Theorem to simple geometric problems. (Given a rectangle that is 4 ft by 7 ft determine the length of the diagonal.)

SAMPLE FUNDAMENTAL CONCEPT EXAM I

1. Completely simplify $\frac{1}{\frac{1}{3}\left(1-\frac{2}{3}\right)}$
2. Solve $7^x = \frac{1}{49}$ for x
3. Expand the product $(3x-4)(5-2x)$ and combine like terms.
4. Find all roots of $x^2 - 10x + 23 = 0$.
5. Find the distance between the points $(1,6)$ and $(6,18)$.
6. Solve $x^2 + 3x \leq 4$ for x.
7. Find an equation of the line that passes through the points $(-1,0)$ and $(2,6)$.
8. If the independent variable of $w(\theta) = 2\theta^2$ is restricted to values in the interval [2,5], what is the interval of all possible values of the dependent variable?
9. Completely simplify $2\cot x \sec x \sin x$.
10. Completely simplify $\left(\frac{3a^{-2}}{b}\right)^{-2}$.
11. Solve $\begin{cases} 3x = 2 - 7y \\ y = x - 5 \end{cases}$ for x and y.
12. Given $g(t) = 2 + 3t$ and $h(s) = 4s^2$, evaluate $g(3) - h(2)$.
13. Graph $y = \sin x - 2$ on the domain $[0, 2\pi]$. Label the axes.
14. If you double the length of the sides of a cube, by how much is the volume affected?
15. Completely simplify $\frac{3}{(\log_4 8 + \log_4 2)}$.
16. Find an equation of the line that is parallel to the line $-4x + 8y = 12$ and passes through the point $(1,1)$.
17. Solve $x^2 + 7x - 8 = 0$ for x.
18. Completely simplify $\left(\sqrt[3]{xy^6}\right)^6$.

19. Given the right triangle below, what is $\tan\theta$?



20. Solve $3 + x > -(x - 2)$ for x.

SAMPLE FUNDAMENTAL CONCEPT EXAM II

1. Write 42 as a $product$ of its prime factors.

2. Write $\sqrt[3]{x^2}$ without using radical notation.

3. If Joe runs the first mile of the APFT in 5 minutes 40 seconds, and he wants to finish the 2-mile course in less than 12 minutes 40 seconds, at what speed must he run the remaining mile?

4. Solve $|x - 5| = 7$ for x.

5. Find the equation of the line that passes through the points $(-2,1)$ and $(-3,1)$.

6. What is the range of the function $h(x) = -x^2$?

7. Solve $\begin{matrix} y = 2x - 5 \\ y^2 = 4x^2 + 5 \end{matrix}$ for x and y.

8. If the hypotenuse of a 30-60-90 right triangle measures 6 inches, how long is the side of the triangle that is opposite from the $30°$ angle?

9. Solve $4 = \log_3 x$ for x.

10. Given that $f(x) = \sin x$, write the function that shifts $f(x)$ to the right by 3 units.

11. Write $6.371\ x\ 10^{-5}$ in decimal notation.

12. Expand the product $(x - 5)(5 - x)$.

13. Solve for the roots of $g(x) = x^2 - 3x - 10$.

14. Solve $x^2 - 5x + 3 > 5 - 4x$ for x.

15. Find the equation of the line that pass through the point $(1,2)$ and that is parallel to the line $y = -3x + 4$.

16. Evaluate $j(x) = \sqrt{x^2 + 5}$ at $x = a$.

17. What is the domain of the function $m(\theta) = \sin(3\theta)$?

18. Completely simplify $\log_3 a + \log_3 b$.

19. Sketch a graph of $y = x^3 + 3$. Label 3 points on your sketch.

20. How long is the diagonal of a 3" x 5" rectangle?

SAMPLE FUNDAMENTAL CONCEPT EXAM III

1. Write $5{,}226{,}000$ in scientific notation.

2. Completely simplify $\sqrt[4]{16u^{12}}$. Assume all variables represent positive numbers.

3. Expand the product $(3x+7)(3x-7)$ and combine like terms.

4. Find all roots of $x^2 + 8x = 1$.

5. Find the distance between the points $(-2,-1)$ and $(-6,-9)$.

6. Solve $x^2 - 5x \leq -4$ for x.

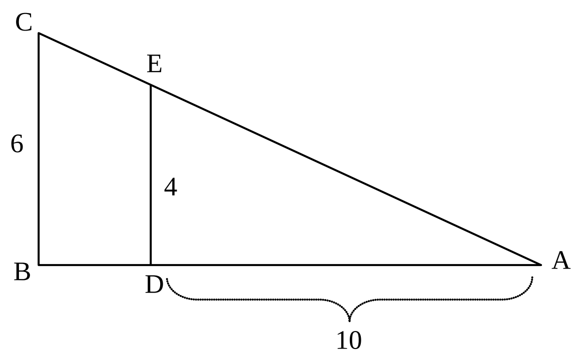
7. What is the range of the function $h(x) = -x^2$?

8. Solve $\begin{matrix} y = 2x - 5 \\ y^2 = 4x^2 + 5 \end{matrix}$ for x and y.

9. If the hypotenuse of a 30-60-90 right triangle measures 6 inches, how long is the side of the triangle that is opposite from the $30°$ angle?

10. Solve $4 = \log_3 x$ for x.

11. Given the triangle below with side DE parallel to side BC and lengths in $feet$, what is the length of side BD?



12. If the diagonal of a rectangle measures 6 $inches$, and its short side measures 2 $inches$, what is the area of the rectangle?

13. Sketch the graph of $y = x^2 - 4$. Label three points on the graph.

14. Find the equation of the line that passes through the points (3,4) and (5,4).

15. What is the domain of the function $h(x) = \sin x$?

16. Given $g(t) = 6t - 2$ and $h(s) = 5s^3$, find $g(h(-1))$.

17. Completely simplify $2\cot x \sec x \sin x$.

18. Write 42 as a product of its prime factors.

19. Solve $|x - 5| = 7$ for x.

20. Completely simplify $\log_4 36 - \log_4 9$.

FUNDAMENTAL DERIVATIVES EXAM

The Fundamental Derivatives Exam (FDE) is administered during the second semester in MA104 to assess a student's knowledge of the rules associated with taking derivatives of elementary single-variable functions - exponential, logarithmic, trigonometric and power functions. Students are expected to calculate the derivatives of these functions (without using technology) using the properties of the derivative (constant multiple, sum and difference rules), power rule, product rule, quotient rule, and chain rule. Each student's success in MA104, as well as subsequent math, science, and engineering courses, depends on a firm understanding of these fundamental differentiation rules. Students must earn an 80% or higher on the FDE in order to be considered for advancement to the next core mathematics course. Students who do not earn an 80% or higher on the first FDE will have the option to retake the exam.

The following is a sample FDE that is similar to an actual exam. Students are given 40 minutes to complete 20 problems without the use of technology.

Find the indicated derivatives.

1. $f(x) = 3x^4 + 7x^2 - 4x$, find $f'(x)$.	6. $h(x) = 5\tan x$, find $\frac{dh}{dx}$.
2. $g(y) = 6e^y$, find $g'(y)$.	7. $p(y) = 2\sqrt{y}$, find $p'(y)$
3. $y(x) = -2\ln\ x$, find $\frac{dy}{dx}$.	8. $f(x) = (2x^3 - 5x)^4$, find $\frac{d}{dx}f(x)$.
4. $r(\theta) = 4\sin\theta$, find $\frac{d}{d\theta}r(\theta)$.	9. $k(x) = 7e^{3x^4-2}$, find $k'(x)$.
5. $r(t) = \frac{\cos t}{2}$, find $r'(t)$	10. $g(s) = \ln(3s^2 + 5)$, find the first derivative with respect to the variable *s*.
11. $y(t) = 2\sin(6t - 2)$, find $y'(t)$.	16. $b(z) = \frac{3z-2}{7z+1}$, find $\frac{d}{dz}b(z)$.
12. $f(x) = \tan(7x^3 - 3x)$, find $f'(x)$.	17. $p(x) = \frac{2x^2-5x+7}{2e^x}$, find $p'(x)$.
13. $h(x) = (2x+1)(x^2-3)$, find $\frac{dh}{dx}$.	18. $h(x) = \frac{4\sin x}{5x-6}$, find $h'(x)$.
14. $h(t) = 2e^t\cos t$, find $h'(t)$.	19. $j(x) = (2x^3+7)^3e^{5x}$, find $j'(x)$.
15. $y(x) = (2x^3 - 4x + 7)\ln x$, find $\frac{dy}{dx}$.	20. $r(t) = \frac{2\cos(3t)}{4\ln(t^2)}$, find $r'(t)$.

HISTORY OF THE DEPT OF MATHEMATICAL SCIENCES AT USMA

A teacher affects eternity; he can never tell where his influence stops. -- Henry Adams

The Department of Mathematical Sciences, USMA, has a rich history of contributing to the education of cadets as confident problem solvers and of developing its faculty as effective teachers, leaders, and researchers. The story of mathematical education at West Point is full of interest: faculty curriculum developments, teaching methods and tools, and technological equipment. Many of the Department's advances have been exported outside the Academy to be employed by other civilian and military educational institutions.

EARLY BEGINNINGS: The actual teaching of mathematics at West Point dates from even before the Academy was established. In 1801, George Baron taught a few Cadets of Artillery and Engineers some of the fundamentals and applications of algebra. The Academy at West Point was instituted by act of Congress and signed into law by President Thomas Jefferson on 16 March 1802. The first acting Professors of Mathematics were Captains Jared Mansfield and William Barron. They taught the first few cadets topics in algebra, geometry, and surveying.

CONTRIBUTIONS TO THE NATION: Since the Academy was the first scientific and technical school in America, the early mathematics professors at USMA had the opportunity to make significant contributions not only to the Academy, but also to other American colleges. Perhaps the most prominent contributors were the early 19th century department heads Charles Davies and Albert E. Church. The work of these two professors had a significant impact on elementary schools, high schools, and colleges across the country. Davies became the Professor of Mathematics in 1823. He was one of the most prolific textbook authors of his day, writing over 30 books from elementary arithmetic to advanced college mathematics. His books were used in schools throughout the country from grade school to college. He had tremendous influence on the educational system of America throughout the 19th century. Albert Church succeeded Davies in 1837, and served as Department Head for the next 41 years. Another influential author, he published seven college mathematics textbooks.

PRODUCING LEADERS FOR THE NATION: Faculty from the Department have been notable military leaders for the country. Robert E. Lee was a standout cadet-instructor in the Department, Omar Bradley served as an Instructor for four years, Harris Jones and William Bessell were Deans of the Academic Board at USMA for a total of 15 years, and Department Heads Harris Jones, William Bessell, Charles Nicholas, John Dick, and Jack Pollin served impressively during two world wars.

The unique technical curriculum in place at the Academy during the middle of the 19th century produced many successful mathematicians and scientists for the country at large. West Point graduates Horace Webster, Edward Courtenay, Alexander Bache, James Clark, Francis Smith, Richard Smith, Henry Lockwood, Henry Eustis, Alexander Stewart, and William Peck filled positions as professors of mathematics or college presidents at other schools such as the U. S. Naval Academy, Geneva College, University of Virginia, University of Pennsylvania, University of Mississippi, Yale, Brown, Harvard, Columbia, Virginia Military Institute, Cooper Institute, and Brooklyn Polytechnic Institute. Two mathematics department heads became college presidents after leaving USMA; Alden Partridge founded and became the first president of Norwich University, and David Douglass served as president of Kenyon College in Ohio for four years. Jared Mansfield was appointed surveyor-general of the Northwest Territory, and Ferdinand Hassler became superintendent of the United States Coastal Survey. Capable individuals such as these exported the West Point model of undergraduate mathematics education throughout the nation.

While the faculty at USMA has been primarily military, the Department has benefited from civilian visiting professors since 1976. As part of the goal for civilianization of 25% of the faculty by 2002, begun in 1992, the Department established in 1994 a Center for Faculty Development in Mathematics. This Center establishes faculty development models and curricula and provides for the development of the "Davies Fellows", who serve as rotating civilian faculty members.

HISTORICAL AND LIBRARY HOLDINGS: Sylvanus Thayer's first task before assuming the Superintendancy in 1817 was to tour the technical institutions of Europe and assess what features USMA could use to advantage. One of Thayer's many accomplishments was to obtain numerous mathematics and science books from Europe. Thayer's book collection included many of the finest books available at that time. His books provided a solid foundation for the USMA library to build upon. Today, the West Point Library has one of the finest collections of pre-20th century mathematics books in the world. Also during the middle of the 19th century, the Academy instructors used elaborate physical models made by Theodore Oliver to explain the structures and concepts of geometry. This magnificent collection of string models is still in the Department today.

CURRICULAR DEVELOPMENT: After Thayer studied the military and educational systems of Europe, he reorganized the Academy according to the French system of the Ecole Polytechnic. The Department of Mathematics faculty (which included as Professor the distinguished scientist and surveyor Andrew Ellicott, and the famous French mathematician Claude Crozet whom Thayer recruited during his European trip to bring to USMA and America his expertise in Descriptive Geometry, advanced mathematics, and fortifications engineering) combined the French theoretical mathematics program with the practical methods of the English to establish a new model for America's program of undergraduate mathematics. This program of instruction in Mathematics grew over several decades and was emulated by many other schools in the country. The initial purpose of the Military Academy was to educate and train military engineers. Sylvanus Thayer, the "Father of the Military Academy" and Superintendent from 1817-1833, instituted a four-year curriculum with supporting pedagogy to fulfill this purpose. Thayer's curriculum was very heavy in mathematics; from Thayer's time to the late 1800's, cadets took the equivalent of 54 credit hours of mathematics courses. The topics covered in these courses were algebra, trigonometry, geometry, descriptive geometry (engineering drawing), analytic geometry, and calculus. Over the years, the entering cadets became better prepared and fewer of the elementary subjects were needed. During Davies' tenure (1823-37), calculus was introduced as a requirement for all cadets, and was used in the development of science and engineering courses. The time allotted for the mathematics curriculum decreased to 48 credit hours by 1940, and to 30 credit hours by 1950. During the 1940's, courses in probability & statistics and in differential equations were introduced into the core curriculum and a limited electives program was started for advanced students. In the 1960's, department head Charles Nicholas (previously one of the organizers of the Central Intelligence Agency) wrote a rigorous and comprehensive mathematics textbook (the "Green Death") that cadets used during their entire core mathematics program. With this text, he was able to adapt the mathematics program to keep up with the increasing demands of modern science and engineering. In the 1970's, Academy-wide curricular changes provided opportunities for cadets to major in mathematics.

During the 1980's, a mathematical sciences consulting element was established that allowed faculty members and cadets to support the research needs of the Army. This type of research activity continues today in the Army Research Laboratory (ARL) Mathematical Sciences Center of Excellence and in the Operations Research Center (ORCEN). In 1990 the Department introduced a new core mathematics curriculum that included a course in discrete dynamical systems, with embedded linear algebra. In that same year, the department changed its name to the Department of Mathematical Sciences to reflect broader interests in applied mathematics, operations research, and computation.

TECHNOLOGICAL DEVELOPMENT: USMA has a long history of technological innovation in the classroom. It was Crozet and other professors at USMA in the 1820's who were the first professors in the nation to use the blackboard as the primary tool of instruction. In 1944, the slide rule was issued to all cadets and was used in all plebe mathematics classes. During William Bessell’s tenure (1947-1959), the mathematics classrooms in Thayer Hall (a converted riding stable) were modernized with overhead projectors and mechanical computers. Bessell was also instrumental in establishing a computer center at West Point. The hand held calculator was issued to all cadets beginning in 1975, and pre-configured computers were issued to all cadets beginning in 1986. In recent years, the department has established a UNIX workstation lab, an NSF-funded PC lab, and has run experimental sections with notebook computers and with multimedia.

NATIONAL COMPETITION: In the spring of 1933 West Point entered an interesting competition in mathematics. After USMA defeated Harvard at a football game the previous fall, a chance remark from President Lowell of Harvard to Superintendent Jones of USMA led the two schools to arrange a mathematics challenge match between the two schools with the two competing teams each composed of 12 second-year cadets. Army was the home team, so the Harvard competitors traveled by train to West Point. All the competitors took a test written by the president of the Mathematics Association of America. The **New York Times**, which had promoted the event with a series of articles in its sports section, reported the results. The West Point "mathletes" defeated Harvard in the competition that was the precursor to the national Putnam

Competition. Since its inception in 1984, the Academy has entered two three-person teams in the International Mathematics Competition in Modeling. USMA won the top prize in 1988, 1993, 2000, 2001, 2002, 2003, and 2004.

During the first half of the 1990's, the Department of Mathematical Sciences at USMA became recognized as one of the more progressive mathematics programs in the country. The Department developed a strong "7 into 4" program that is exciting as well as innovative. Throughout the 1990's and into the 21st century, the Department of Mathematical Sciences has led the nation in curricular reform and in nurturing interdisciplinary cooperation and collaboration between and among partner disciplines. Project INTERMATH, a National Science Foundation grant to develop integrated curriculum and ILAPs, assisted in supporting these efforts. The USMA mathematics program has had great influence on mathematics education in America throughout its history, and strives to continue contributing to the improvement of mathematical education in America.

For a more extensive exposition on the history of the Department, refer to http://www.westpoint.edu/math/SitePages/Detailed History.aspx.