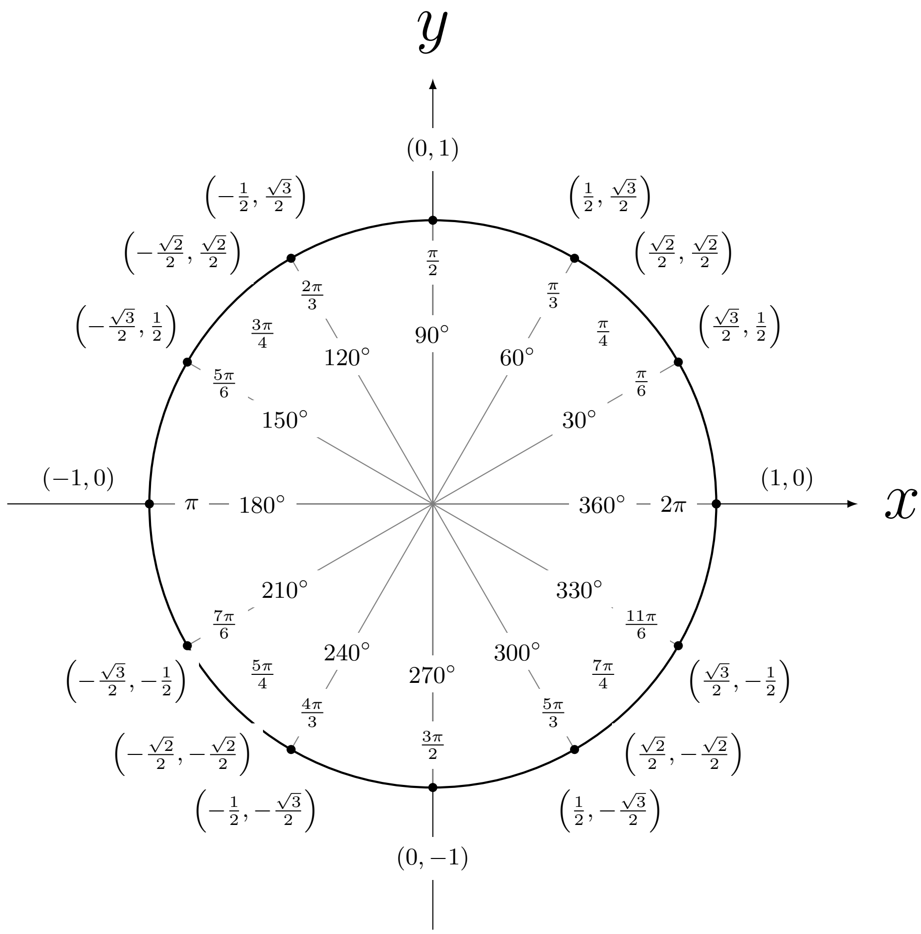
Trigonmetry

The Unit Circle:



– This is a circle of radius one. The prefix uni- means one. Thus, the unit circle has a radius equal to one.

– The equation for the unit circle is $x^2 + y^2 = 1$.

– The circumference of any circle is $2\pi r$, thus the cicumference of the unit circle is 2π.

– $\frac{1}{4}$ of the distance around the unit circle is $\frac{\pi}{2}$

– $\frac{1}{2}$ of the distance around the unit circle is π

– $\frac{3}{4}$ of the distance around the unit circle is $\frac{3\pi}{2}$

– the full distance around the unit circle is 2π

NOTE: Any point on the unit circle has a coordinate (x, y). If we draw a right triangle from the origin, $(0,0)$, to the point on the unit circle, (x, y), to the point on the x-axis, $(x, 0)$, we can use the following formulas for sine, cosine, and tangent to show that for any point on the unit circle that:

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\tan\theta = \frac{y}{x}$$

Match four of the following functions to the graphs below; then, graph the remaining two functions.

$a.\ \ f(x) = 1 + \sin x \quad b.\ \ g(x) = 1 - \sin x \quad c.\ \ h(x) = 3\sin x$

$d.\ \ r(x) = \cos 2x \quad e.\ \ s(x) = 3\sin(x) \quad f.\ \ m(x) = \sin 2x$

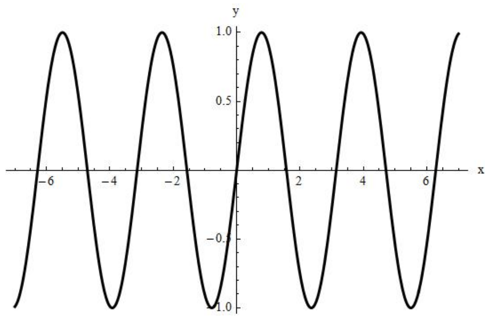


Figure 1: $m(x) = \sin 2x$

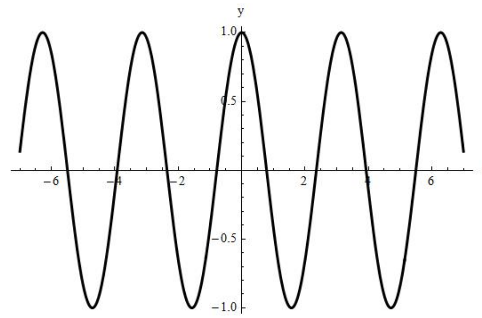


Figure 2: $r(x) = \cos 2x$

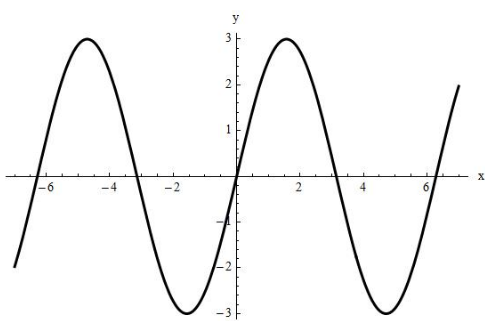


Figure 3: $s(x) = 3\sin x$

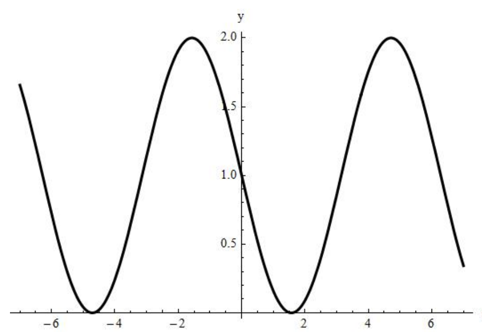


Figure 4: $g(x) = 1 - \sin x$

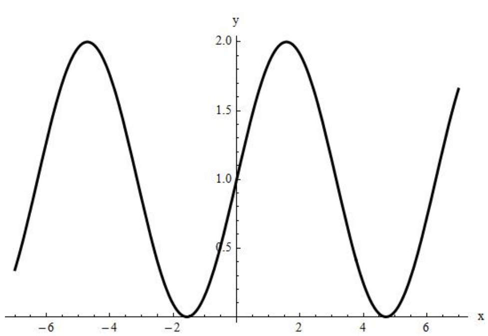


Figure 5: $f(x) = 1 + \sin x$

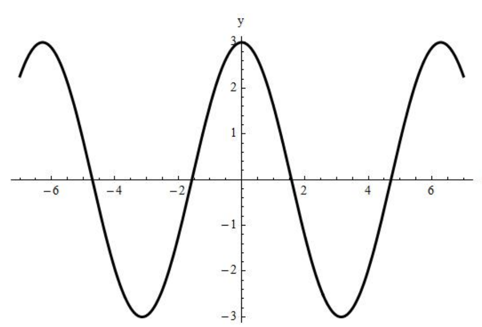


Figure 6: $h(x) = 3\sin x$

Radians and Degrees

Conversions:

$1\,radian\,(rad) = \left(\frac{180}{\pi}\right)\; degrees\,(^\circ)$
$\pi\; radians = 180\; degrees$
$1\; degree = \left(\frac{\pi}{180}\right)\; degrees$

1. Find the radian measure of the angle when given the degree measure:

a. $36^\circ = 36^\circ\left(\frac{\pi\, rad}{180^\circ}\right) = \frac{\pi}{5}\, rad$ *b.* $200^\circ = 200^\circ\left(\frac{\pi\, rad}{180^\circ}\right) = \frac{10\pi}{9}\, rad$ *c.* $45^\circ = 45^\circ\left(\frac{\pi\, rad}{180^\circ}\right) = \frac{\pi}{4}\, rad$
d. $-72^\circ = -72\left(\frac{\pi}{180}\right) = -\frac{2\pi}{5}\, rad$ *e.* $60^\circ = 60\left(\frac{\pi}{180}\right) = \frac{\pi}{3}\, rad$ *f.* $115^\circ = 115\left(\frac{\pi}{180}\right) = \frac{23\pi}{36}\, rad$
g. $-135^\circ = -135\left(\frac{\pi}{180}\right) = -\frac{3\pi}{4}\, rad$ *h.* $150^\circ = 150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}\, rad$ *i.* $-420^\circ = -420\left(\frac{\pi}{180}\right) = -\frac{7\pi}{3}\, rad$

2. Find the degree measure of the angle with the following radian measure:

a. $\frac{3\pi}{4}\, rad = \frac{3\pi}{4}\left(\frac{180}{\pi}\right) = 135^\circ$ *b.* $-\frac{7\pi}{2}\, rad = -\frac{7\pi}{2}\left(\frac{180}{\pi}\right) = -630^\circ$ *c.* $\frac{5\pi}{6}\, rad = \frac{5\pi}{6}\left(\frac{180}{\pi}\right) = 150^\circ$
d. $-\frac{\pi}{12}\, rad = -\frac{\pi}{12}\left(\frac{180}{\pi}\right) = -15^\circ$ *e.* $-1.5\, rad = -1.5\left(\frac{180}{\pi}\right) = -\frac{270}{\pi}^\circ$ *f.* $\frac{2\pi}{9}\, rad = \frac{2\pi}{9}\left(\frac{180}{\pi}\right) = 40^\circ$
g. $\frac{\pi}{5}\, rad = \frac{\pi}{5}\left(\frac{180}{\pi}\right) = 36^\circ$ *h.* $\frac{\pi}{18}\, rad = \frac{\pi}{18}\left(\frac{180}{\pi}\right) = 10^\circ$ *i.* $\frac{5\pi}{3}\, rad = \frac{5\pi}{3}\left(\frac{180}{\pi}\right) = 300^\circ$

Trigonometric Identities

Simplify the following trigonometric expressions:

1. $(\sin\theta)^2 + (\cos\theta)^2 - 1 = 0$

2. $(\sin\theta + \cos\theta)^2 + 2\cos\theta = 2\sin\theta\cos\theta + 2\cos\theta$

3. $(\sin\theta)(\cos\theta) + (\sin\theta)^3 - 2 = \sin\theta - 2$

4. $2(\cos\theta)^2 + 2(\sin\theta)^2 + 1 = 3$