

USEFUL FORMULAS FOR USMA MATHEMATICAL RECALL KNOWLEDGE

The following constitutes a basic mathematical vocabulary that will be built upon during each cadets four-semester core mathematics experience and in his or her future science/technology/engineering/mathematics (STEM) courses. Once each of these basic ideas has been covered in class, each cadet can be required to reproduce, upon demand in any future lesson of any STEM course, that idea exactly as shown here. Annotated beside each heading or item is the course number in which the cadet is responsible for each item. These items are recall knowledge - cadets are also required to be proficient in the more conceptual, less-verbatim ideas and skills reflected in each core math courses Course Objectives section of this document. The items listed in the sections from **Algebra** through (and including) **Properties of Functions** are the mathematical skills represented on the **Fundamental Concepts Exam (FCE)**.

ALGEBRA (MA100 / MA103)

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|---|--|
| 1. $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | 2. $a^b \cdot a^c = a^{b+c}$ |
| 3. $(ab)^c = a^c b^c$ | 4. $\frac{a^b}{a^c} = a^{b-c}$ |
| 5. $y = \log_b x \Rightarrow x = b^y$ | 6. $\log_b (b^x) = b^{\log_b (x)} = x$ |
| 7. $\log_b x^a = a \log_b x$ | 8. $\log_b ac = \log_b a + \log_b c$ |
| 9. $\log_b \frac{a}{c} = \log_b a - \log_b c$ | 10. $\log_b a = \frac{\log_c a}{\log_c b}$ |

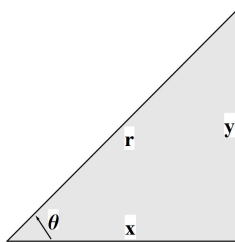
ANALYTIC GEOMETRY (MA100 / MA103)

Rectangle:	Area = lw	Perimeter = $2l + 2w$
Circle:	Area = πr^2	Circumference = $2\pi r$
Rectangular Solid:	Volume = lwh	Surface Area = $2lw + 2lh + 2hw$
Cylinder:	Volume = $\pi r^2 l$	Surface Area = $2\pi r^2 + 2\pi r l$
Sphere:	Volume = $\frac{4}{3}\pi r^3$	Surface Area = $4\pi r^2$

Distance between (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

TRIGONOMETRY (MA100 / MA103)

With reference to the right triangle:

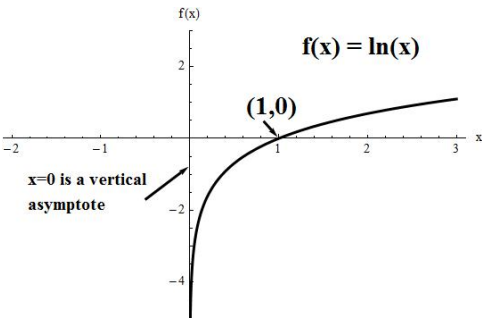
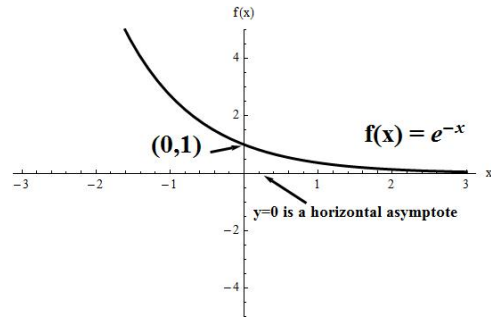
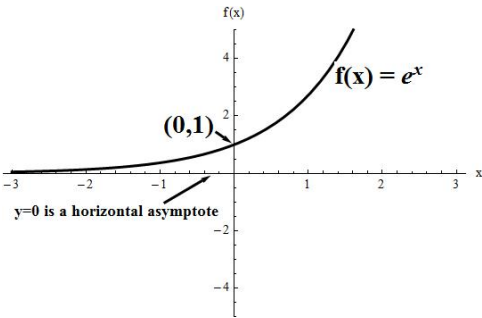
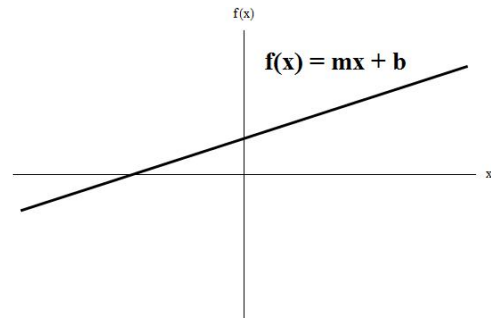
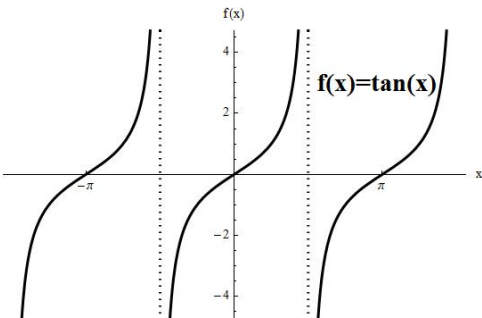
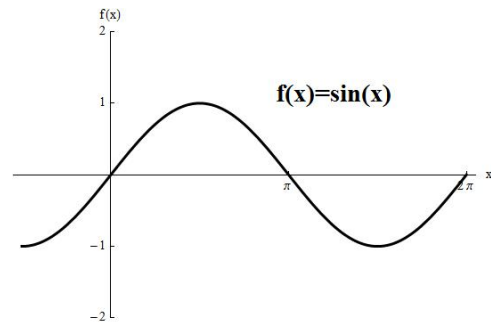
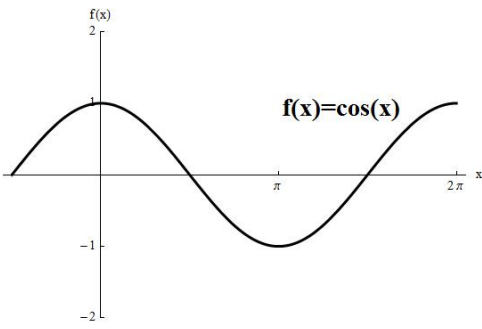


$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$	$2\pi \text{ radians} = 360 \text{ degrees}$
$\cot \theta = \frac{1}{\tan \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\csc \theta = \frac{1}{\sin \theta}$	
$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$x^2 + y^2 = r^2$	$\sin^2 \theta + \cos^2 \theta = 1$	

RELATIONSHIPS (MA100 / MA103)

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| 1. Corresponding sides of similar triangles are proportional | 2. Distance = average rate \times time |
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PROPERTIES OF FUNCTIONS (MA100/MA103)



DIFFERENTIATION (MA104)

1. $\frac{d}{dx}(a) = 0$
2. $\frac{d}{dx}(x) = 1$
3. $\frac{d}{dx}(au) = a\frac{du}{dx}$
4. $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ (Product Rule)
6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (Quotient Rule)
7. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$ (Power Rule)
8. $\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)]\frac{du}{dx}$ (Chain Rule)
9. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$
10. $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$
11. $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$
12. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$

INTEGRATION (MA104)

13. $\int adx = ax + C$
14. $\int (u+v) dx = \int u dx + \int v dx$
15. $\int x^n dx = \frac{x^{n+1}}{n+1} + C (n \neq -1)$
16. $\int e^{ax} dx = \frac{e^{ax}}{a} + C$
17. Understand and be able to apply the Substitution Rule
18. $\int \frac{du}{u} = \ln|u| + C$
19. $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$
20. $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$

Fundamental Theorem of Calculus:

21. If f is integrable on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$ where $\frac{dF}{dx} = f(x)$

VECTOR CALCULUS (MA103/MA104/MA205)

22. $|\vec{A}| = \sqrt{a_i^2 + a_j^2 + a_k^2}$
23. $\vec{A} \bullet \vec{B} = a_i b_i + a_j b_j + a_k b_k = |\vec{A}||\vec{B}| \cos \theta$
24. $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \theta$
25. $\nabla f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k}$
26. $\vec{A} \times \vec{B} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix} = (a_j b_k - a_k b_j) \hat{i} - (a_i b_k - a_k b_i) \hat{j} + (a_i b_j - a_j b_i) \hat{k}$

PROBABILITY AND STATISTICS (MA206)

27. PDFs and CDFs:

Discrete: PDF: $P(X = x) = p(x)$
CDF: $P(X \leq x) = \sum_{y \leq x} p(y)$

Continuous: PDF: $f(x)$ is used to find probabilities of the form $P(a \leq X \leq b) = \int_a^b f(x) dx$

CDF: $F(X) = P(X \leq x) = \int_{-\infty}^x f(y) dy$

28. The total accumulation of a probability distribution function is 1.

Discrete: $\sum p(x) = 1$ Continuous: $\int_{-\infty}^{\infty} f(x) dx = 1$

29. Calculate and interpret the expected value (mean) of a random variable.

Discrete: $E(X) = \sum x \cdot p(x)$ Continuous: $E(X) = \int_{-\infty}^{\infty} x \cdot f(x) dx$

30. Calculate and interpret the variance of a random variable.

The variance is an expected value: $V(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$

31. Percentiles of random variables including the median (50th percentile).

32. Central Limit Theorem: Let X_1, X_2, \dots, X_n be a random sample from any population with a finite mean and variance. Then, if n is sufficiently large, $\bar{X} = \sum \frac{X_i}{n}$ and $T_0 = \sum X_i$ are **approximately** normally distributed.